

THE BULLETIN OF THE



USER GROUP

+ CAS-TI

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A Non Polynomial System of Equations |

- [1] **Einführung in die Matrix-Algebra / Mit Anwendungen in der Statistik**, *Karsten Schmidt & Götz Trenkler*, Springer
- [2] **Wiskunde voor hoger onderwijs door middel van Derive 1 & 2**, *Peter van der Velden et al.*, Academic Service – SDU, The Hague, NL
- [3] **Experimente und Visualisierung**, mit Beiträgen von *G. Schmidt, E. Lehrmann, J. Böhm.*, MU Der Mathematikunterricht, Jg 52, Heft 4-2006, Erhard Friedrich Verlag, Seelze
- [4] **Dynamisch Geometrie entdecken**, Elektronische Arbeitsblätter für die 7. – 10. Klasse, *H.-J. Elschenbroich u. Günter Seebach*, coTec-verlag
- [4] and many other valuable books (German and English) are available from

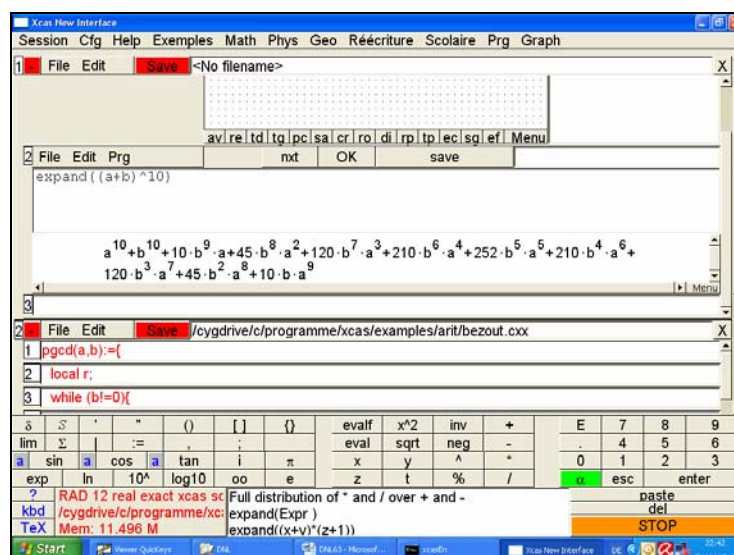
<http://shop.bk-teachware.com>

Recommended websites:

For your information:

<http://www-fourier.ujf-grenoble.fr/~parisse/english.html>

Daniel Legendre



Nils Hahnfeld was busy as ever ...

GREAT NEWS: Calculus Made Easy version 8.0 is now available.

Calculus Made Easy v8.0 has become the ultimate educational tool to study Calculus.

No other software provides detailed Step by Step Solutions to Integration (using U-Substitution (choose u yourself or let CME do it for you!), Integration by Parts, by Partial Fractions, Power Rule, Expand/Rewrite, Arctan(x) and Arcsin(x) Integrals, etc) and Differentiation (Chain-, Power-, Product- and Quotient Rule) problems.

Just enter the given function and learn how the correct rule is applied - step by step - until the final answer is derived.

No other software covers Calculus I (Calc AB), II (Calc BC), III (multivariable), Differential Equations, Vector Calculus and Differential Geometry!!

To view some examples, check here: www.ti89.com/documentation.htm

Dear DUG-Members,
with some delay and with many I present DNL#63. I must admit that I intended to skip DNL#63 and intended to have a double issue 63/64 in December but then I prepared a paper for a seminar – the nonlinear regression – and from one moment to the other I wanted to put it into a smaller than usual DNL#63. As you can see, it did not luck. This issue is again a full publication. Many mails from and to Peter Schofield, Karsten Schmidt, Bernhard Kutzler, Tania Koller and Bill Wilburn were exchanged and I am happy that the 44 pages are again full of materials.

In this DNL you might have your first meeting with the new TI-CAS device and software TI-Nspire. Bernhard Kutzler gives a short introduction. If anybody of you has some experience with Nspire, he or she are friendly welcome to share it with us. The developers announce that as many DERIVE features as possible will be implemented into Nspire. I hope that they will have enough inspiration to do it!

Several DUG-members asked if we could not open our publication for other computer algebra systems. Of course, we can and we start with our first special CAS guest, *MATHEMATICA*. Bill Wilburn sent a respective contribution. It might be nice for students to reproduce calculation and graph with DERIVE (or the TI!). So if you have any article in Maple, MuPAD, Mathematica, Auto-graph (which is not a CAS, but a very fine math program) etc, then please send it to me.

I complained for years that DERIVE is unable to plot in different line thicknesses. Tania Koller shows us to overcome this deficiency. The “trick” is so easy and I am very disappointed that it was me who found it. Tania’s students are very eager using thick lines and

also combining DERIVE and a graphic program – which is also very novel. (I am quite sure that many of you had never tried this). Tania sent a collection of Halloween-Ghosts which were created by applying polynomial functions. They are all descendants of Paul Drijver’s ghost – Hi Paul, do you remember Krems 1993, where you presented your ghost? Your ghost feels very well and has survived in our regions (maybe because of the excellent wines of Krems and surroundings?)

Admire the creativity of Tania’s students on pages 6 and 14.

One contribution is redemption of my promise of DNL#59 to produce contour plots in DERIVE. This is in a tight connection with implicit plots and so we can have implicit plots with thick lines. Thanks Peter Schofield for your great cooperation. It was just yesterday that Peter sent his improved tool for numerically solving equations which is used for calculating the level curves. (More about this in DNL#64.)

Some interesting requests are remaining for the next DNL (Slider bars and Arrows, a very fine tool to collect coordinates of 2D-points in the Algebra window, a TI-bug?, ...).

I received a fine article from G P Speck, New Zealand, who rejoined the DUG, welcome again.

Finally I’d like to invite you to order the Proceedings CD of the DES-TIME Conference. It is full of materials (papers, programs, files, pictures, ...)

Best regards until December



Download all *DNL-DERIVE*- and TI-files from

<http://www.austromath.at/dug/>

<http://www.derive-europe.com/support.asp?dug>

The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE & CAS-TI User Group*. It is published at least four times a year with contents of 44 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE* and the *TI-89/92/Titanium/Voyage 200* as well as to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

As many of the *DERIVE* Users are also using the *CAS-TIs* the *DNL* tries to combine the applications of these modern technologies.

Contributions:

Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE & CAS-TI Newsletter* will be.

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Next issue: December 2006
Deadline 31 November 2006

Preview: Contributions waiting to be published

Two Stage Least Squares, M. R. Phillips, USA
Some simulations of Random Experiments, J. Böhm, AUT
Wonderful World of Pedal Curves, J. Böhm
Another Task for End Examination, J. Lechner, AUT
Tools for 3D-Problems, P. Lüke-Rosendahl, GER
ANOVA with *DERIVE & TI*, M. R. Phillips, USA
Financial Mathematics 4, M. R. Phillips
Hill-Encryption, J. Böhm
Farey Sequences on the *TI*, M. Lesmes-Acosta, COL
Simulating a Graphing Calculator in *DERIVE*, J. Böhm
Henon & Co, J. Böhm
Challenges from Fermat, Bj. Felsager, DEN
Are all Bodies falling equally fast, J. Lechner
Modelling Traffic Density, Th. Himmelbauer, AUT
Do you know this? Cabri & CAS on PC and Handheld, W. Wegscheider, AUT
An Interesting Problem with a Triangle, Steiner Point, P. Lüke-Rosendahl, GER
Mathematics and Design, H. Weller, GER
Diophantine Polynomials, D. E. McDougall, Canada
Challenger Matrix Problems, G P Speck, New Zealand
Precise Recurring Decimal Notation, P. Schofield, UK
and Setif, FRA; Vermeylen, BEL; Leinbach, USA; Koller, AUT; Baumann, GER;
Keunecke, GER,and others

Impressum:

Medieninhaber: *DERIVE* User Group, A-3042 Würmla, D'Lust 1, AUSTRIA
Richtung: Fachzeitschrift
Herausgeber: Mag. Josef Böhm
Herstellung: Selbstverlag

Three mails from Bill Wilburn:

I am glad to hear that the DNL will continue on. A while back, when I heard that Dr. Rich and Theresa were no longer with Derive, I suspected that this might happen. For a long while, I've been interested in Mathematica. So, I downloaded the trial version. I liked it. My wife got me Mathematica 5.2.0 Professional for Father's Day. I'm still a novice with it. It has so many features. It's amazing, and you can do so much with it. I personally like all of the CAS systems. I've had no experience with Maple, but it appears to be a fine CAS system as well. I hope that you can get the latest version of Mathematica. I know that a person of your caliber certainly deserves it.

Sincerely,
Bill Wilburn

PS. I'm looking forward to more issues of the DNL.

Hi Josef:

Here's an example of a Mathematica Notebook on solving 2 sets of parametric equations.
I used DERIVE 6.10 as a word file to show the Mathematica Notebook's contents.
I suppose that you could put it into a DNL if you wish. If so, please feel free to edit it as you like.

Have a nice day,
Bill Wilburn

Hi Josef:

I ran across this site and thought that you might be interested:

<http://curvebank.calstatela.edu/famouscurves/famous.htm>

Regards,
Bill Wilburn

You can find Bill's example on page 38 together with the respective DERIVE treatment, Josef

Manuel Garcia Mateos

garcia_mateos@WEB.DE

Hello Derivians,

I tried to show that the sequence $(1 + \frac{1}{n})^n < 3$ for all natural numbers by defining n as a positive integer and solving the inequality.

Derive does not resolve the inequality for n. Derive produces the following output:

#1: $n \in \text{Integer} (0, \infty)$

#2: $\text{SOLVE}\left(\left(1 + \frac{1}{n}\right)^n < 3, n\right) = n \cdot \text{LN}\left(\frac{n+1}{n}\right) < \text{LN}(3)$

Is there a way to show that the sequence is bounded and monotonously increasing without using Bernoulli-inequality or other transformations?

Regard the sequence $e(n) = 3 \cdot (-0.7)^n$ (n positive Integer) The sequence is bounded and converges to zero. but: $\text{solve}(e(n) < 5, n)$ leads to false!

Is there a way to show that alternating sequences are bounded?

Greetings,
Manuel

G P Speck, Wanganui, New Zealand

Dear Josef:

It was good to receive your email and pleasing to hear that you remember Wanganui fondly.

For various reasons I'm having difficulty sending technical material via email – so for this one time I am sending a disc with the Derive V.5 file ch_0(Challenger).dfw containing my paper *Challenger Matrix Problems* which you viewed previously. With this disc you can, of course, modify the format however you wish for sending to DUG members. Not being familiar with Derive V.6 until I receive my copy, I hope that this will be no problem.

In the meantime I'll try to get my difficulties on margins, matrices, email formats, and other printing matters sorted out.

Keep up the good work with Derive – we'll hope that most of the good features of Derive will be maintained in NSpire.

Sincerely,

G P Speck

Many thanks to G P. The diskette arrived in Würmla and its contents will make a fine article for a new ACDC-column. Josef

Peter Schofield, UK

Hello Josef,

Thank you for your news letter informing me of the end of TI's interest in Derive. I'm quite upset about it, but I suppose it was coming from the time they decided not to employ Albert anymore. I have strong reservations about "Nspire" and what it is intending to achieve. I fear that many of the wonderful aspects of Derive will be lost.

Anyway, I am attaching a short article - together with corresponding Derive and TI-89 files - on how a precise recurring decimal notation can be set up in Derive 6 and on the TI-89. I hope this might be suitable for the DUG?

All the best

Peter

Peter's contribution is on my stack and will be published in the next DNL.

Peter Witthinrich, Germany

Lieber Josef Böhm,

schade, dass die Entwicklung von Derive eingestellt wird. Derive ist in meinem Umfeld eher akzeptiert worden als die CAS-Rechner, ich habe es im Unterricht deutlich häufiger als die Kleinrechner eingesetzt.

Hoffentlich übersieht TI nicht, dass so gut wie alle Schulen ohnehin einen Computerraum haben, sich aber nicht alle Schulen zusätzlich noch weitere Rechner anschaffen können.

Außerdem gibt es inzwischen eine Anzahl weiterer Computeralgebrasysteme, auch wenn Derive aus meiner Sicht besonders (oder besser als die anderen) für den Schulbereich geeignet ist. Manche Lehrkräfte steigen vielleicht einfach um, zumal einige CAS kostenlos sind oder die Grundversion (die in vielen Fällen für den Unterricht ausreicht) kostenfrei ist.

Meistens hat bisher der Elternverein die kleinen Rechner bezahlt. Jetzt werde ich den Eltern klarmachen müssen, dass nach der noch nicht so lange zurück liegenden Anschaffung der Voyage200 schon wieder ein neuer Typ besorgt werden soll.

Zur DUG denke ich, dass sie doch ohnehin bereits auf CAS-Rechner erweitert war. So sehe ich keinen Grund, sie nicht bestehen zu lassen. Falls erwünscht könnte sie später immer noch erweitert oder umbenannt werden.

Mit besten Grüßen
Peter Witthinrich

Lisa Townsley, USA, sent a short answer wrt my report on the "(non) Future" of DERIVE in my DUG-Information. I am very happy these three sentences. Many thanks on behalf of the DUG, Lisa. We will go on.

Josef,
This was a nice, sensitive note to send. I have shared your words with my department. The best part about using Derive is the excellent colleagues all over the globe.

I sent Karsten Schmidt my paper about the "nonlinear" regression (page 25) and he sent back an extended and interesting answer. I try to translate his mail (JB.dfw and the pdf-file can be downloaded).

Dear Josef,

in the meanwhile I dealt with your mail from, in particular with your first example ($n = 10$). (page 25).

The extremal value problem $\Phi(x) = a + b \cdot \sin(x) + c \cdot \cos(x)$ is a linear regression with 3 exogene variables (including a pseudo variable for the intercept). Even if $\sin(x)$ and $\cos(x)$ are created by a nonlinear transformation, the regression model remains linear.

This can easily be observed by inspecting your matrix M, which corresponds to matrix X' in chapter 12 of our textbook (published by Springer, for an example see page 218)¹. I reproduced the procedure in DERIVE using the "usual" notation (file JB.dfw). #3 creates the regressor matrix X from x.; KQ-estimation with X and y leads exactly to your result. #7 and #8 allow a 2D-scatter plot together with the regression line. (Comment: I believe that KQ is "kleinste Quadrate" = "least squares".)

The 2. attachment is the pdf-SPSS output.

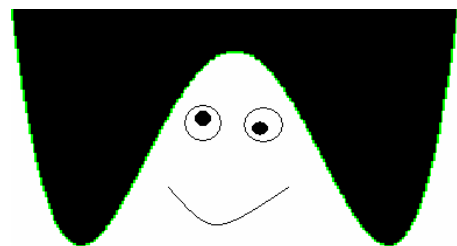
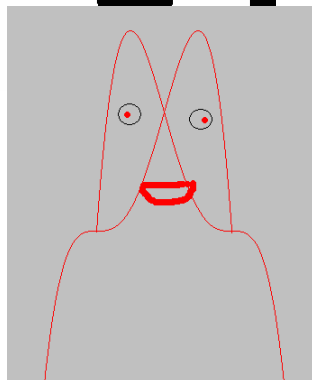
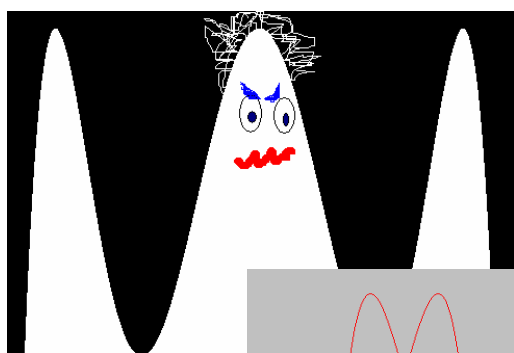
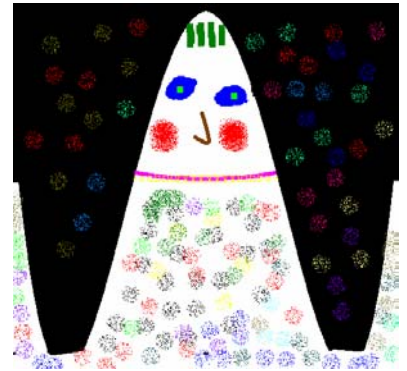
1. page: Scatterplot and "linear" regression;
2. page: nonlinear regression with model equation $a + b \cdot \sin(x) + c \cdot \cos(x)$, i.e. the model from above with transformed variables, which leads to the same results;
3. page: nonlinear regression with model equation $a \cdot \sin(b \cdot x + c) + d$, which is equivalent to the SinReg-treatment of the TI-92+/Voyage 200, which gives the same result as in your paper. also entsprechend dem Ansatz SinReg des TI-92, es kommt auch das raus wie in deinem Text.

Under "Residual"/"Sum of Squares" you can find in SPSS the respective SSE - sum of squared errors – (twice 1.772 and once 1.645), which shows that the (true) nonlinear regression fits better.

By the way, this is a fine demonstration of Gauß-Markow-Theorem which says that the KQ-estimator (LS-estimator) is the best (i.e. minimizing variance) of all linear unbiased estimation function, because the better estimator is a NONlinear one! Best regards - Karsten

¹ see our book shelf.

Tania Koller's students like thick lines, colours,
mathematics and **Halloween**



Thick Lines and Colour Pots in DERIVE

Tania Koller, Austria

My colleague Tania gave a great presentation in Dresden (on the Proceedings CD). Among many other “motivating activities” for students she showed her “Cinderella’s Shoe” modelled by cubic splines. I have seen this earlier, but look at her “DERIVE picture” of the shoe: thick lines and a bright pink, which is not among the DERIVE shading colours. I asked how she produced thick lines in DERIVE which has been a long requested feature of DERIVE, and how she brought the colour on the shoe. Here is her answer with some additional comments. Josef



$$\#1: f(x) := \frac{x^4 - 13x^2 + 30}{30} - 2$$

$$\#2: \text{TABLE}(f(x), x, -5, 5, 0.01)$$

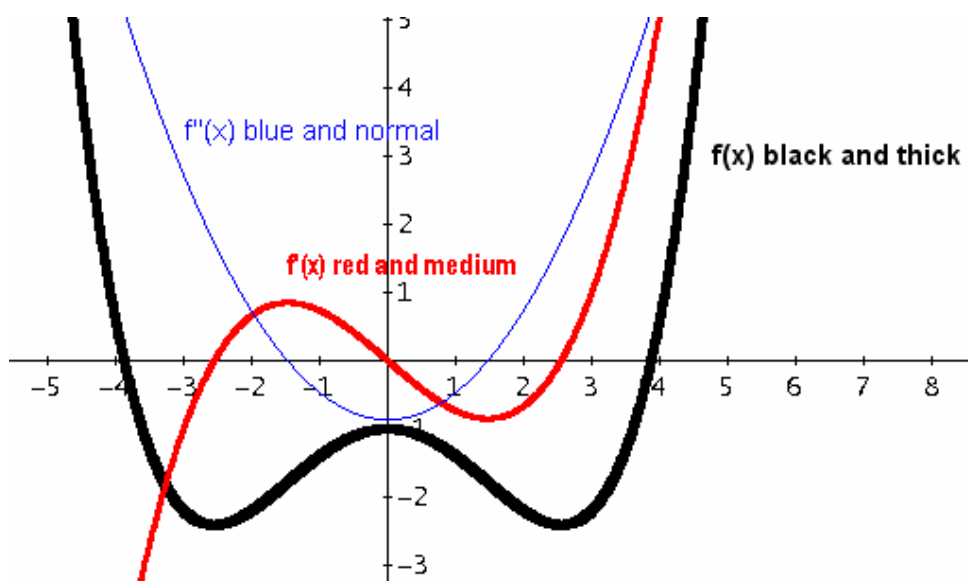
$$\#3: \text{TABLE}(f'(x), x, -5, 5, 0.01)$$

$$\#4: f''(x)$$

Set Option > Display > Points Connected and Large and Plot #2 (don't forget to set Options > Approximate before plotting).

Then Set Option > Display > Points connected and Medium and Plot #3.

Finally plot #4. Choose colours as appropriate.

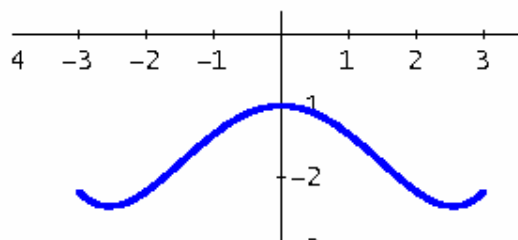


If you intend to use this nice tool more often then you might use a function with fixed parameters which can be changed if needed.

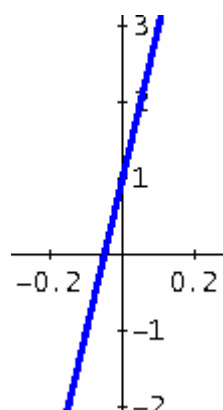
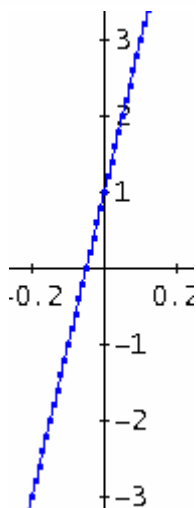
In function `thick()` variable and the increment of the table are preset with `x` and `0.01`. In most cases these defaults will be fine – and you need only entering function and boundaries:

#5: `thick(u, mi, ma, v_ := x, inc := 0.01) := TABLE(u, v_, mi, ma, inc)`

#6: `thick(f(x), -3, 3)`



#7: `thick(20·t + 1, -0.5, 0.5, t)` #8: `thick(20·t + 1, -0.5, 0.5, t, 0.001)`



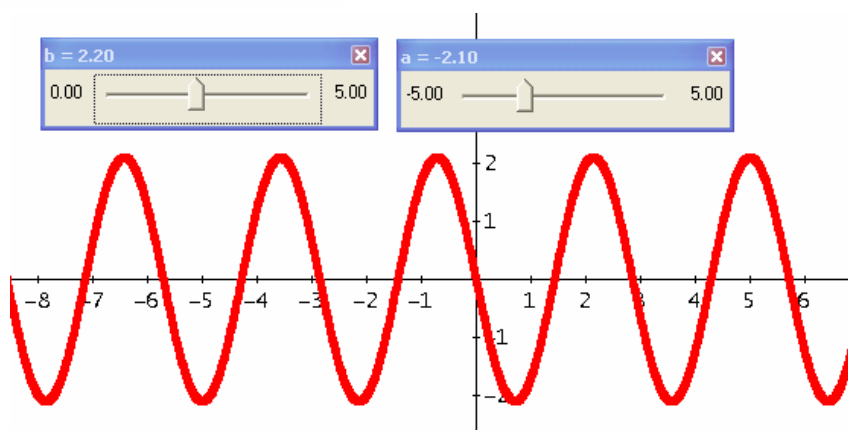
Now it looks fine.

There is no Anti-Aliasing-algorithm – so set the points as tight as possible.

I enter variable t and see that the steepness of the graph needs a smaller increment.

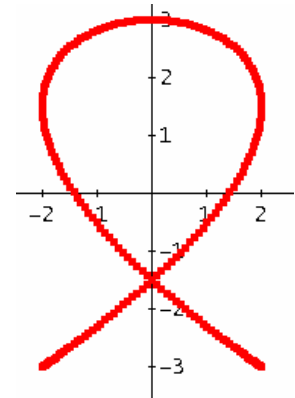
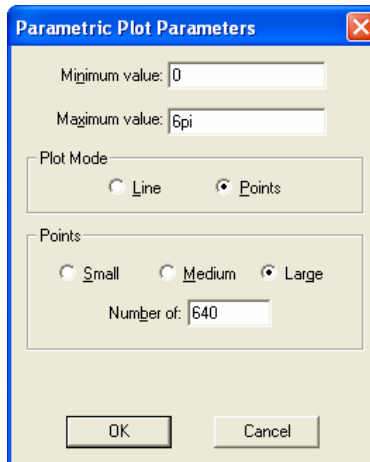
The slider bar is one of my favourite features of DERIVE 6 (and as I know, it is also very often used by Tania in classroom). So I am very happy that the "Thick- and Medium-Line-Trick" is wonderful supported by the slider bars. Sometimes it needs some moments to create the graph, but then the slider bars work without any delay.

#9: `thick(a·SIN(b·x), -4·π, 4·π)`



In parameter form we do not need this trick, because we can enter the size of points:

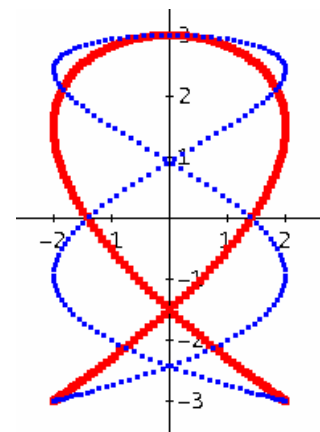
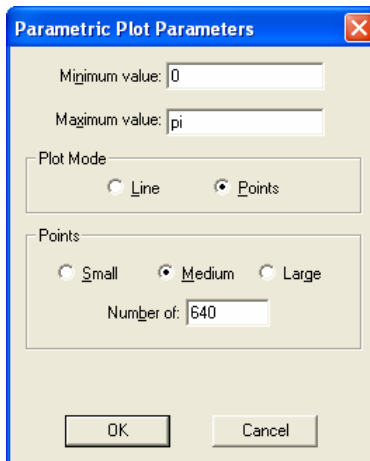
#10: $[2 \cdot \sin(3 \cdot t), 3 \cdot \cos(2 \cdot t)]$



Plotting in Size Medium we face problems because of the restriction to plot a maximum number of 640 points. To obtain a smooth line one has to plot in intersections $0 - \pi, \pi - 2\pi$, etc.

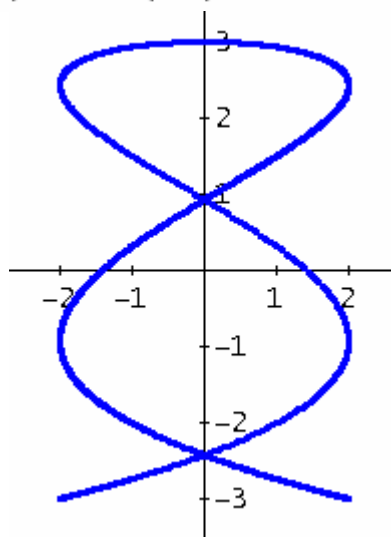
#11: $[2 \cdot \sin(5 \cdot t), 3 \cdot \cos(2 \cdot t)]$

Or we try to overcome this restriction by defining a function applying the more or less unrestricting VECTOR-command:



#12: `thickpar(u, mi, ma, v_ := x, inc := 0.01) := VECTOR(u, v_, mi, ma, inc)`

#13: `thickpar([2 * sin(5 * t), 3 * cos(2 * t)], 0, 3 * pi, t)`



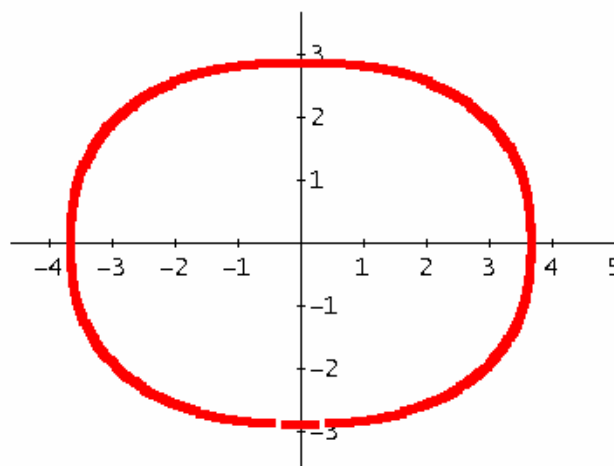
Inspired and inspired by Tania's method I wanted to do implicit plots in three line thicknesses, but this is not so easy to perform. One idea is to adapt the utility for plotting level curves (see another contribution in this DNL). I tried another way: Scanning the area in small x- and y-increments and plotting the points which fulfill the implicitly given equation of the curve:

```

impl(u, xst, xend, yst, yend, inc := 0.01, acc := 0.01, pts, i, j, val) :=
  Prog
  pts := []
  u := LHS(u) - RHS(u)
  i := xst
  Loop
  If i > xend
  RETURN pts
  j := yst
  Loop
  If j > yend exit
  If ABS(SUBST(u, [x, y], [i, j])) < acc
  pts := APPEND(pts, [[i, j]])
  j := j + inc
  i := i + inc

```

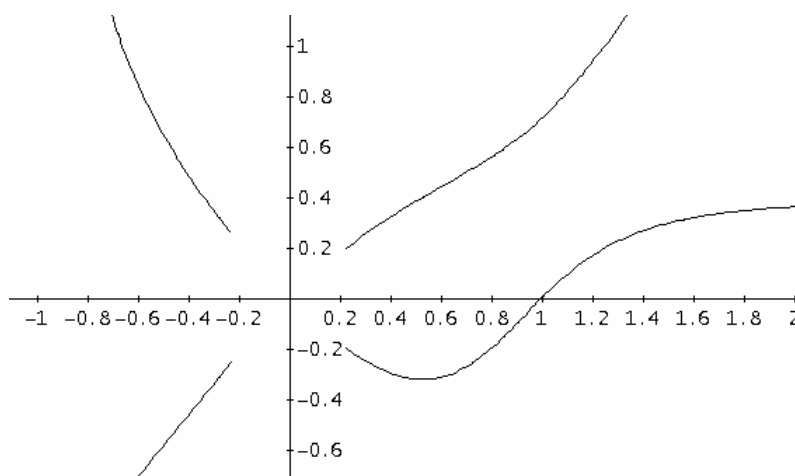
#17: $\text{impl}\left(\left|\frac{x}{3.67}\right|^{2.37} + \left|\frac{y}{2.88}\right|^{2.37} = 1, -4, 4, -4, 4, 0.03\right)$



(I'll come back to this "Superellipse" in the next DNL, Josef)

I wanted to test my function and invented a strange implicit form and DERIVE gave immediately its graph:

#18:
$$\frac{3 - x \cdot y}{x^2 - y^2} = 1$$

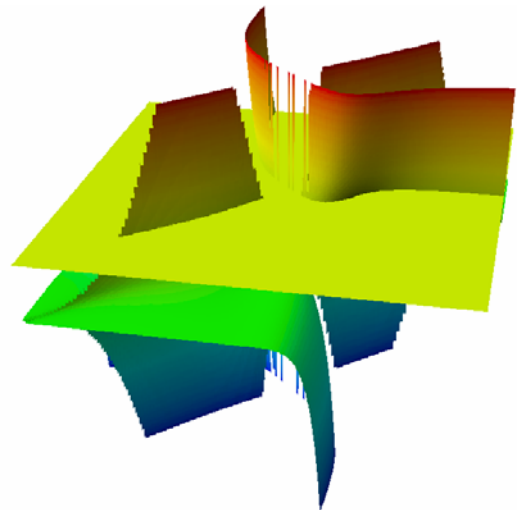
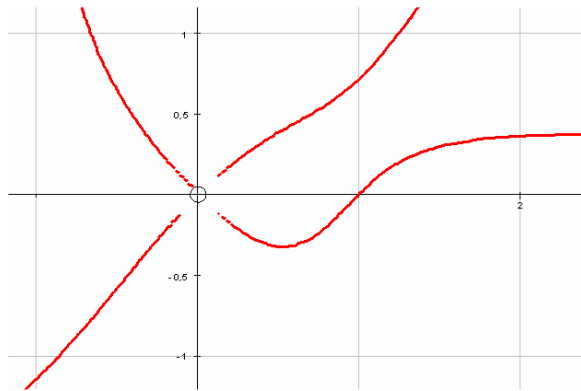


But we want the graph as a medium or as a thick line.

Autograph is not a CAS but has a lot of great features, with a very powerful graphic.

I plotted the expression with a thickness of 3 ½ pts:

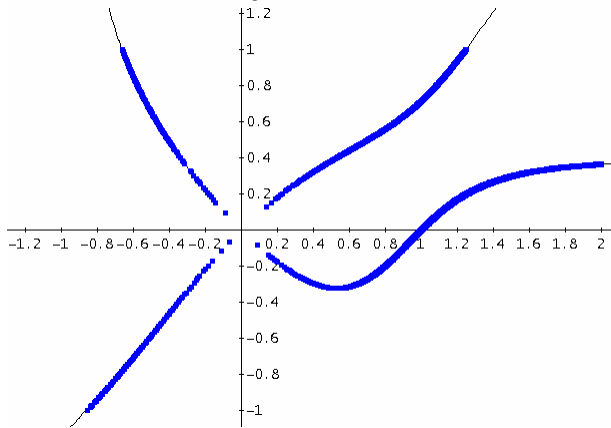
And I took DPGraph to plot the left hand side surface together with the plane $z = 1$ for better illustration:



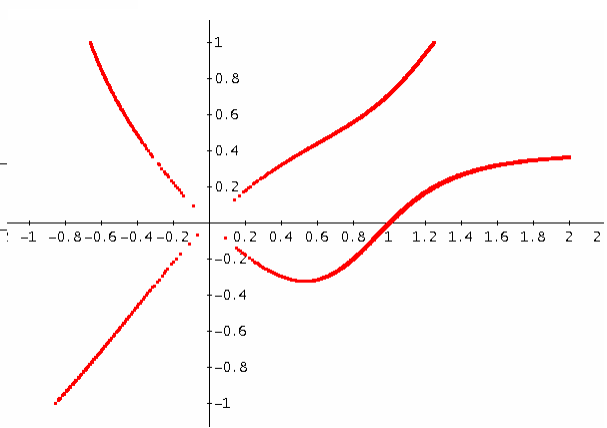
Autograph above and DPGraph at the right hand side...

And the DERIVE – Plots below!

$$\#19: \text{imp1} \left(\frac{\begin{pmatrix} 3 & -x \cdot y \\ x & \cdot e \end{pmatrix}}{\begin{pmatrix} 2 & 2 \\ x & -y \end{pmatrix}} = 1, -1, 2, -1, 1, 0.002, 0.005 \right)$$



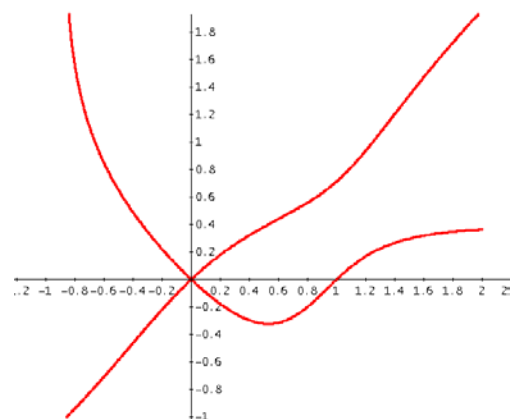
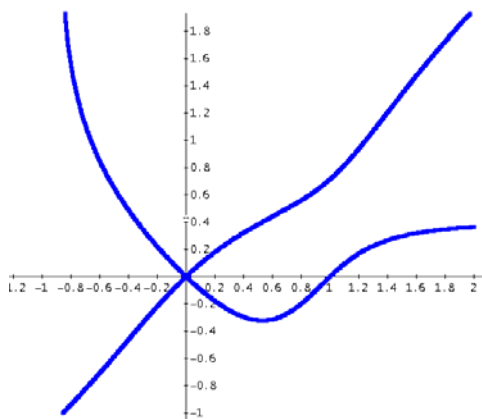
Point Size Large

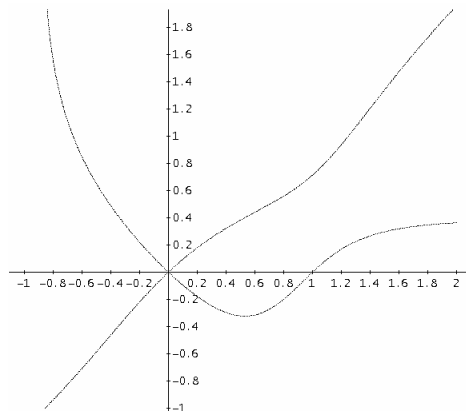


Point Size Medium

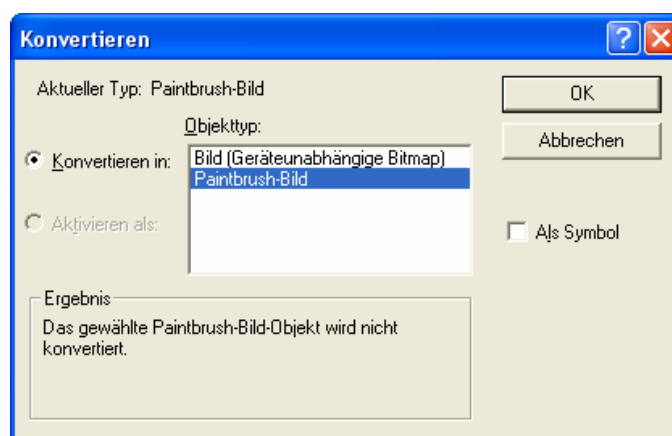
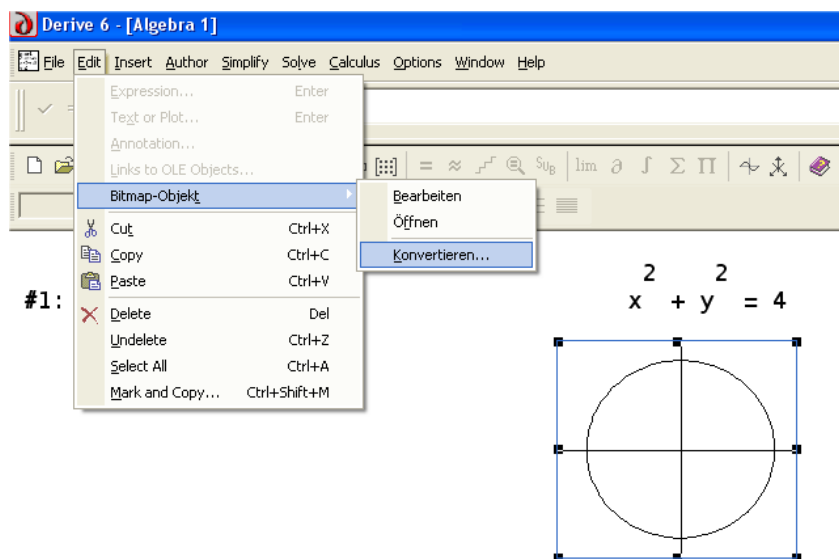
Another possibility is offered by adapting one or the other function of producing contour- or level plots (see the contour plot - contribution in this DNL). If you don't mind calculation time then try the following and plot applying different point sizes:

$$\text{IMPLOT} \left(\frac{\begin{pmatrix} 3 & -x \cdot y \\ x & \cdot e \end{pmatrix}}{\begin{pmatrix} 2 & 2 \\ x & -y \end{pmatrix}} = 1, -1, 2, -1, 2, 0.1, 0.1, 0.1 \right)$$

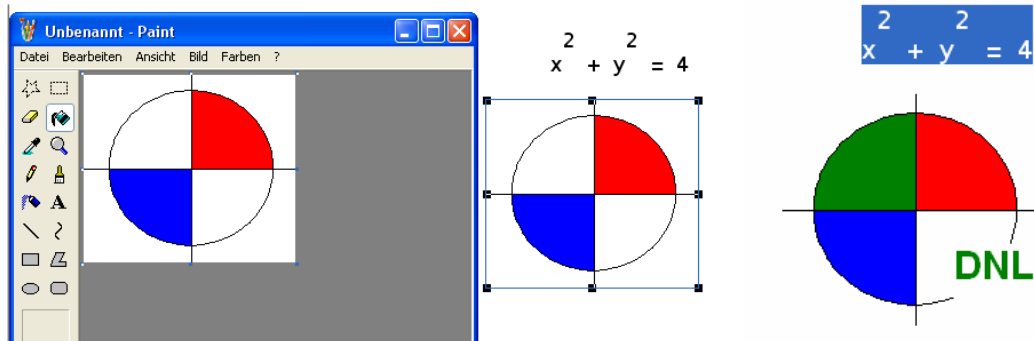




Now let's turn to Painting in DERIVE. Tania told me that one of her students (Eva Maria Korntheuer, 16 years) came across an Option in the Edit-Menu which was never mentioned in class before: Edit > Bitmap-Object. (Have you ever noticed this?). Just for fun she wanted to know what would happen. She converted the graph (eg a circle) into a Paintbrush Picture. And then with a double click on this picture she found herself in the Windows Tool Paintbrush with all its features at her disposal. Many thanks Eva Maria and keep your interest and curiosity. (She created the bear on page 14.)

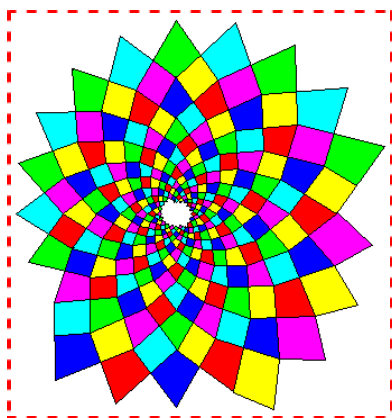


In the Graphics Program you can manipulate the graph – remaining in DERIVE – as you like and clicking in the DERIVE screen (eg on the expression) you see the coloured graph. Any double click on it leads back to Paintbrush for further treatment.



Tania's students created a whole gallery of pictures using various functions. She sent a selection of pictures and I am very happy to present a part of this gallery in the DNL.

I found the graph below in a mathematics book it is a challenge for students to reproduce this pattern.



$$\text{spir} := \left[-0.5 \cdot e^{0.5 \cdot t} \cdot \sin(t), -0.5 \cdot e^{0.5 \cdot t} \cdot \cos(t) \right]$$

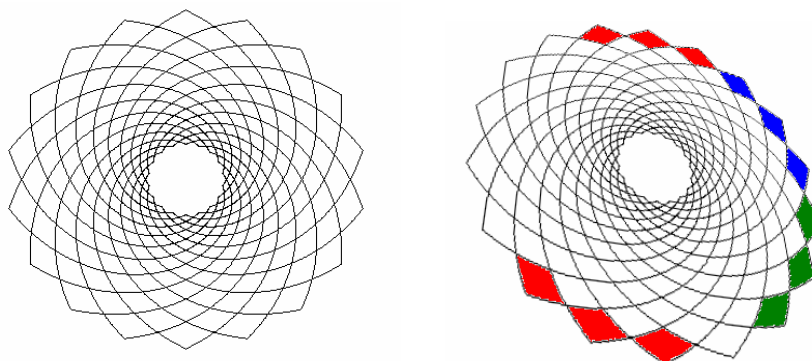
$$\text{spir2} := \left[\frac{e^{t/2} \cdot \sin(t)}{2}, -\frac{e^{t/2} \cdot \cos(t)}{2} \right]$$

$$\text{rot}(s) := \begin{bmatrix} \cos(s) & -\sin(s) \\ \sin(s) & \cos(s) \end{bmatrix}$$

$$\left[\text{VECTOR}\left(\text{spir} \cdot \text{rot}\left(\frac{k \cdot \pi}{9}\right), k, 0, 17\right), \text{VECTOR}\left(\text{spir2} \cdot \text{rot}\left(\frac{k \cdot \pi}{9}\right), k, 0, 17\right) \right]$$

Plot with $0 \leq t \leq \pi$.

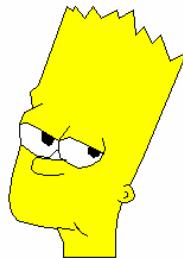
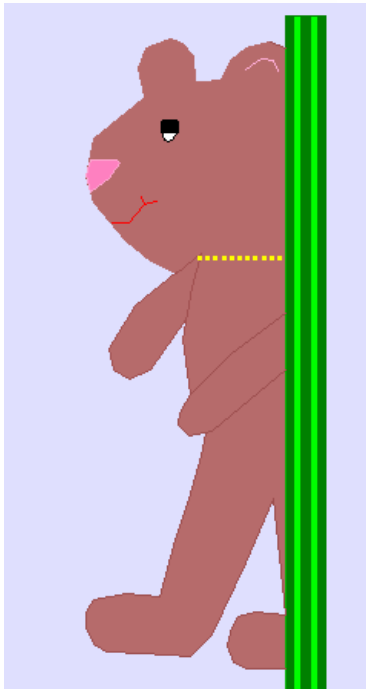
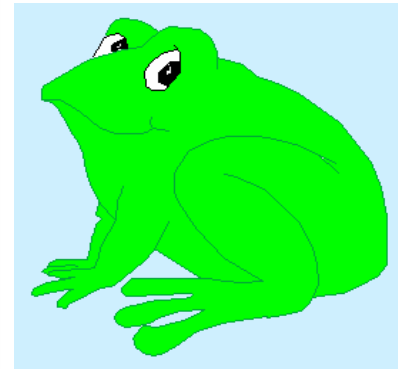
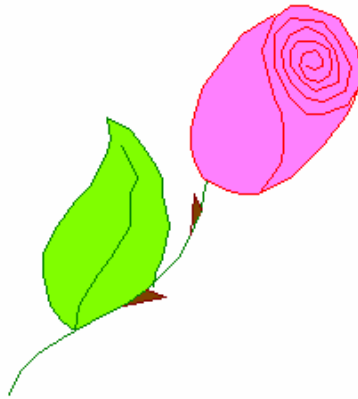
It is not so easy to find the mathematics behind. This is one way to produce a similar pattern by two families of curves. The graph was distorted in Paintbrush and then the tiles could be filled.



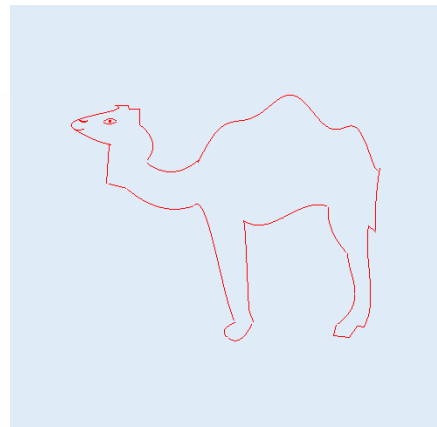
You can change the DERIVE built in colour palette, but the shadings look pale because of the shading algorithm, the Paintbrush colours are really bright colours! And it would be very tiresome to shade the many facets of the pattern above ... and of the graphs on the next page!

Gallery of DERIVE-Graphs treated as Paintbrush-Pictures

(produced by Tania's Students of the Handelsakademie St. Pölten, Lower Austria)



Bart Simpson



The dromedary (by artist Daniel Wailzer) is made up by a list of polynomials up to degree 10!

You can find a lot of other students' products in Tania's great presentation given at DES-TIME 2006 in Dresden on the Conference CD.

Unfortunately Tania did not unhide her secret how to bring colour on the screen of her Voyage 200!! Josef

F2 F4 F5
Trace Regraph Math D



DEG AUTO FUNC

TI-Nspire – The next Generation?

Bernhard Kutzler

Linz, Austria

At this year's Derive conference Texas Instruments and Soft Warehouse Europe announced the discontinuation of Derive. The reason is that TI will soon release their new CAS product TI-Nspire and in the future all development and marketing resources shall be focused on this new product. TI-Nspire carries the technology of the legendary TI-92 further into a new generation Math/CAS tool.

This article is written for people familiar with Derive and presents some TI-Nspire features which are not available in Derive, hence can be regarded an enrichment by a Derive user. This text is based on a pre-release version of the TI-Nspire software (version 1.0.500). The same can be performed on a handheld, on which some of the screen images look slightly different.

There are many features in Derive 6 which are not (yet) available in TI-Nspire. Most of these features are there, because the user community demanded them. These features resulted from the feedback of countless teachers who used earlier versions of Derive in their classrooms and reported their findings and suggestions to the Derive development team. Clearly, if you use such a feature in your classroom, you would miss it in TI-Nspire, in particular if you were among those who asked for it.

There are exciting features in TI-Nspire which we had asked for in Derive, but our wishes were not (maybe could not be) granted. So the ideal would be a union of both products ...

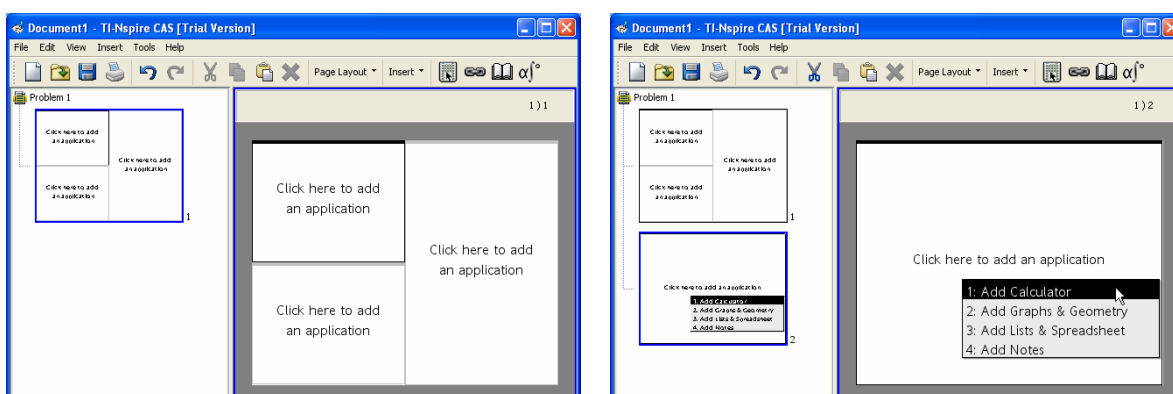
In all areas of life we see technology leaps. From 1967 to 1972 NASA used Apollo rockets for their space flights with Apollo "versions" 1-17. Then they changed to the Space Shuttle program. Today we use cars with combustion engines which burn oil products. In a world with oil getting short, hence getting more and more expensive, we will soon need a "technology upgrade" to a next generation of motors using alternative fuels. Change is the essence of life!

TI-Nspire is an exciting new product. Its modern design has a potential which goes far beyond what is practically possible in Derive. By offering, seamlessly integrated, computer algebra, dynamic geometry, spreadsheet technology, word processing, data collection, and data exchange with a handheld, you now have, "under the same roof", all technologies which are used today in mathematics teaching.

My hope is that many Derive users will look at TI-Nspire, test it in the classroom, compare it to Derive, and help with their feedback to shape TI-Nspire so that it will be not only Derive's declared successor, but will become an upgrade to Derive, hence a true "next generation". When comparing the two products we do need to bear in mind that Derive is an adult, while TI-Nspire is a new kid on the block, quickly growing up in today's (software) world, certainly having a long and exciting future ahead. So please see the potential – and help turning as much as possible into reality.

TI-Nspire

In the opening screen of TI-Nspire the preview pane on the left shows the overall structure of the document. A TI-Nspire document is a collection of “problems”. (An empty document starts with only one problem, named “Problem 1”.) Each problem is a collection of “pages”. Each page can be used as a whole or it can be divided into up to four sub pages. Each page (or sub page) can be connected to one of the following four applications: “Calculator”, “Graphs & Geometry”, “Lists & Spreadsheet”, and “Notes”. Right of the preview pane is the working area, which shows the selected “page”, hence offers an interactive environment for the respective application (or applications, if the page was divided into sub pages).



The left screen image shows a page which was divided into three sub pages, but with no applications connected to them yet. The right screen image shows a second page added to the first page, with a menu open for connecting an application to it.

Calculator / CAS

The “Calculator” application provides the CAS and corresponds to the “Home” screen on a TI-89/92/92+ or Voyage200.

When entering expressions one receives a lot of valuable support. Say, we want to input $x + x + \sin(x)$. After typing $x+x+si$, the following is displayed on the screen:

$$x+x+si$$

Variables are displayed in italics and currently this would be interpreted as the sum of the three terms x , x , and si . As soon as we type the letter n , we see on the screen:

$$x+x+\sin$$

TI-Nspire now recognizes the string \sin as the name of a built-in function, hence does not display it in italics any longer. Adding the letter x leads to:

$$x+x+\sin x$$

Now the third term, $\sin x$, is all displayed in italics, because it is interpreted as the name of a four-letter variable. This is a clear “warning” to the user that this is not the intended input, which was the sine of x , simply because there need to be a pair of parentheses around the argument of the sine func-

tion². We delete the last character, x , with the backspace key and add an opening parenthesis, $($.

TI-Nspire automatically introduces a matching closing parenthesis with the cursor placed in between. Typing x completes the desired input:

The automatic insertion of a matching parenthesis is a great help to avoid “missing parenthesis” errors.

Entering the input with the return key produces a pair of expressions, comprising the entry and the answer, the latter being the (algebraically) simplified version of the entry.

Unlike Derive and like the TI-92/89 or Voyage200, TI-Nspire automatically simplifies every entry.

For entering powers, derivatives, integrals, systems of equations, limits, sums, products, matrices, piecewise defined functions, and certain function logic constructs (such as if-then-else and loops) TI-Nspire offers two-dimensional templates. Say, we want to compute the derivative of $x^2 \ln(x)$. We type x , then hit the keyboard’s power key followed by the digit 2.

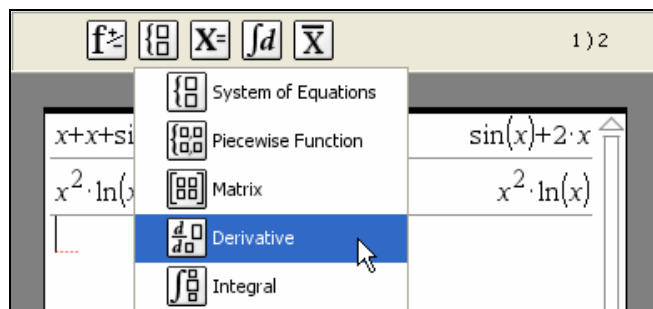
The 2 is displayed as a raised-up exponent. The current position of the cursor indicates that the next character would be entered on the exponent level. The right arrow key exits the exponent level and brings the cursor down to the level of variable x . The tab key would do the same.

Since there is no ambiguity, we may simply type “ $\ln(x)$ ”, i.e. without a multiplication symbol (and also without the closing parenthesis).

For computing the derivative we can choose the corresponding function (and template) from the **Expression Templates** menu or the **Calculus** menu:

² TI-Nspire, like the TI-92/89/Voyage200, does not offer Derive’s Character Input Mode.

We open the **Expression Templates** menu, ...



... from which we choose the **Derivative** template.

$$\frac{d}{dx}(\quad)$$

The red dots under and the vertical cursor left of the left-most (and lowest) dashed rectangle indicate that the next input goes into this position. Obviously this is for the variable. We type x followed by the right arrow key \rightarrow (or the tab key \rightarrow) to move to the next input field.

$$\frac{d}{dx}(\quad)$$

The second input field is for the order of the derivative. If left empty, a first order derivative will be computed. This is what we want, hence we need not input the number one, but can move to the next input field with \rightarrow or \rightarrow .

$$\frac{d}{dx}(\quad)$$

There are several techniques to input expression $x^2 \ln(x)$ as the third argument. Note that it was entered already in the previous step. Hence, we could type `ans` (which is an abbreviation of “answer”) as a reference to the previous answer, i.e. the desired expression. As an alternative we use the up-arrow key \uparrow to highlight the last answer, ...

... then we use the enter key \leftarrow to paste a copy of the highlighted expression at the position where the input cursor was before choosing this expression. Another use of the enter key \leftarrow completes the input.

$$\frac{d}{dx}(x^2 \cdot \ln(x))$$

$$\frac{d}{dx}(x^2 \cdot \ln(x)) \quad 2 \cdot x \cdot \ln(x) + x$$



In general Derive's CAS engine is still more powerful than TI-Nspire's CAS engine, but there are already a few exceptions, such as solving trig equations, ...

$$\text{solve}\left(2 \cdot \cos(x) + (\sin(x))^2 = 1, x\right) \quad x = \frac{(2 \cdot n2 - 1) \cdot \pi}{2}$$

... for which TI-Nspire may offer all solutions (by introducing a parameter, $n2$ in our example).



Recently there has been a debate about how Derive approximates expressions involving inverse trig functions in degree mode. When we use Derive to simplify, everything is fine:

#6:	$\text{ASIN}\left(\frac{1}{2}\right) = 30^\circ$
#7:	$\text{ASIN}\left(\frac{1}{3}\right) = \text{ASIN}\left(\frac{1}{3}\right)$

However, when we approximate these expressions, the resulting angles are given in radians – even if the system is set to degree mode.

#8:	$\text{ASIN}\left(\frac{1}{2}\right)$
#9:	0.5235987755
#10:	$\text{ASIN}\left(\frac{1}{3}\right)$
#11:	0.3398369094

One can obtain degree answers when using ARCSIN instead of ASIN, ...

#12:	$\text{ARCSIN}\left(\frac{1}{2}\right)$
#13:	30
#14:	$\text{ARCSIN}\left(\frac{1}{3}\right)$
#15:	19.47122063

... but this is not satisfactory when solving trig equations.

#1:	$\text{SIN}(x) = \frac{1}{2}$
#2:	Angle := Degree
#3:	$\text{NSOLVE}\left(\text{SIN}(x) = \frac{1}{2}, x\right)$
#4:	$x = -3.665191456$

If the above is a nuisance to you, you probably will find the answers provided by TI-Nspire to your liking.

$\sin^{-1}\left(\frac{1}{2}\right)$	30
$\sin^{-1}\left(\frac{1}{3}\right)$	$\sin^{-1}\left(\frac{1}{3}\right)$
$\sin^{-1}\left(\frac{1}{3}\right)$	19.4712

$\text{solve}\left(\sin(x)=\frac{1}{2},x\right)$	
$x=30\cdot(12\cdot n+5)$ or $x=30\cdot(12\cdot n+1)$	
$\text{nSolve}\left(\sin(x)=\frac{1}{3},x\right)$	19.4712



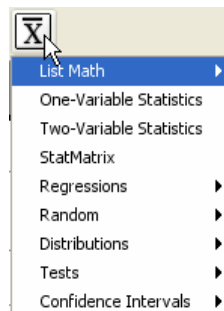
TI-Nspire offers the option to represent complex numbers in either rectangular or polar form:

$\text{cSolve}(x^3=1,x)$	
$x=\frac{-1}{2}+\frac{\sqrt{3}}{2}\cdot i$ or $x=\frac{-1}{2}-\frac{\sqrt{3}}{2}\cdot i$ or $x=1$	

$\text{cSolve}(x^3=1,x)$	
$x=(1 \angle 120)$ or $x=(1 \angle -120)$ or $x=1$	

1

TI-Nspire also offers a statistics menu with a lot of useful functions.



The user interface allows the typical Windows-standard mouse-click-and-drag method to select any part of an expression on the screen.

$x=\frac{-1}{2}+\frac{\sqrt{3}}{2}\cdot i$ or $x=\frac{-1}{2}-\frac{\sqrt{3}}{2}\cdot i$ or $x=1$	
---	--

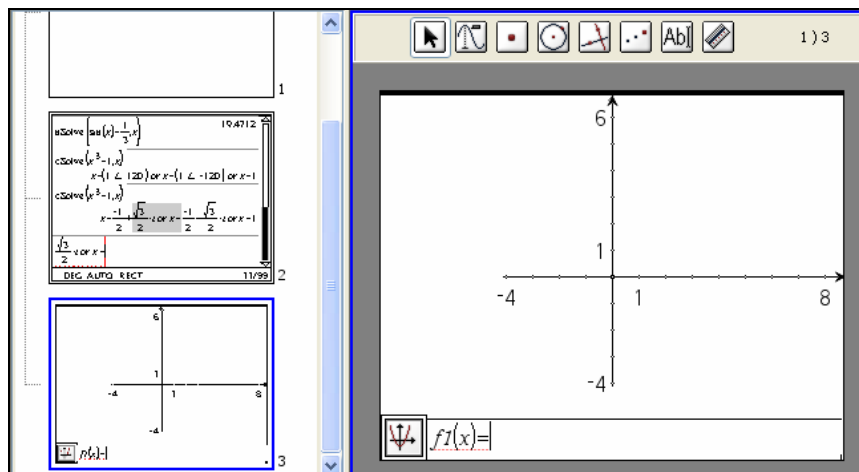
$x=\frac{-1}{2}+\frac{\sqrt{3}}{2}\cdot i$ or $x=\frac{-1}{2}-\frac{\sqrt{3}}{2}\cdot i$ or $x=1$	
---	--

In combination with the usual cut-copy-paste technique this is useful when building new expressions from old ones. However, other than in Derive, the selection of portions of an expression is not restricted to valid subexpressions.

$x=\frac{-1}{2}+\frac{\sqrt{3}}{2}\cdot i$ or $x=\frac{-1}{2}-\frac{\sqrt{3}}{2}\cdot i$ or $x=1$	
$\frac{\sqrt{3}}{2}\cdot i$ or $x=$	

Graphs & Geometry

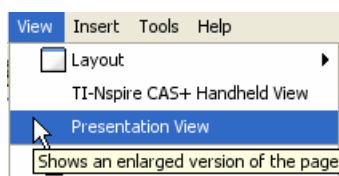
We use the **Insert>Graphs&Geometry** command to add a page for function graphing and interactive dynamic geometry.





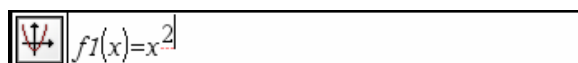
The preview pane on the left now shows a reduced calculator page and a reduced graphing page, both of which are not interactive. One can easily flip between pages (i.e. making them interactive) by clicking the desired page in the preview pane.


In the upper right corner of the application window, next to the application window's icon menu, the string **1)3** indicates that this is Page 3 of Problem 1.

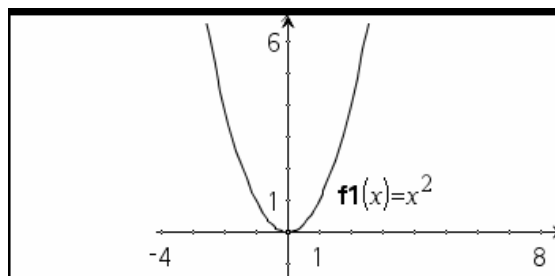
By the way, TI-Nspire offers a **Presentation View** and a **TI-Nspire CAS+ Handheld View**. The first one is very useful if one presents in a bigger room, the second one is very useful when the students use handhelds. In both views the preview pane is not visible.



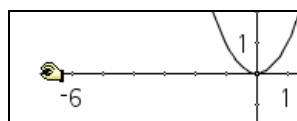
Back to function graphing. To obtain the graph of $y = x^2$, we enter the function term x^2 at the bottom of the graphing window, hence turning the system reserved function name $f1$ into a user-defined function. As it is customary in TI's graphing calculators, $f1$, ..., $f99$ are reserved for this purpose. This technique is similar to producing graphs on a graphing calculator, where one enters the function terms in a "y=" window. In TI-Nspire one sees only one function (and its definition) underneath the graphing window and one can use the up  and down  arrow keys to scroll.



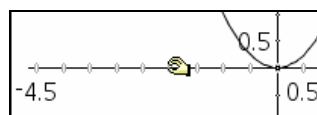
Also here we have the two-dimensional input of expressions. After entering the input with , the graph is drawn and is appropriately labelled with $f1(x)=x^2$.



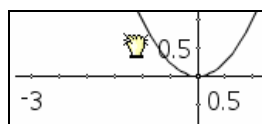
This graphics easily can be changed in many ways simply with the mouse. We choose the **Pointer** tool from the first menu, grab the horizontal axis at its left end (grabbing is performed by moving the mouse to the desired position, then clicking and holding the left mouse button) and extend it further to the left, ...



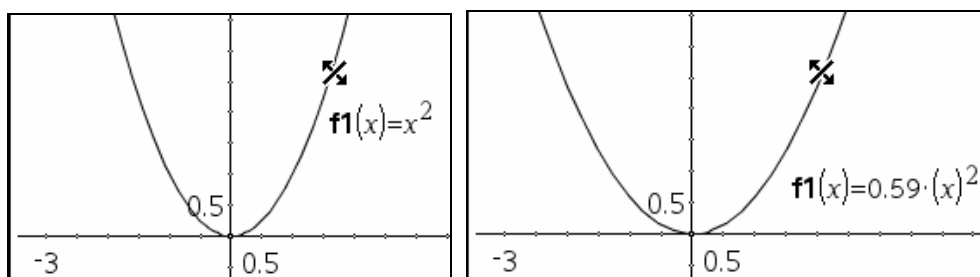
... then we grab the horizontal axes at one of its tick marks and change this axis' scale factor, ...



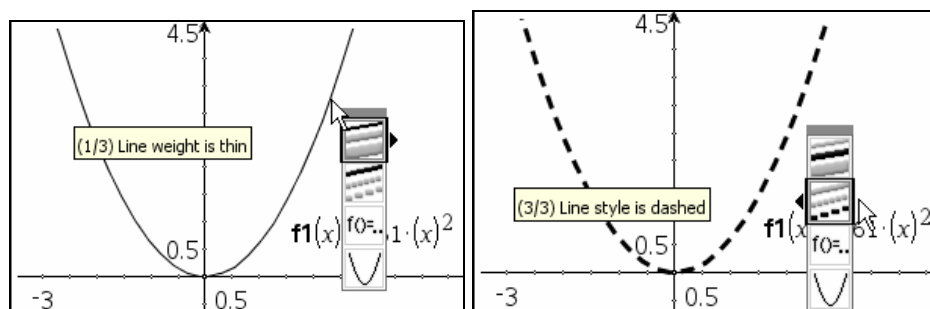
... and finally we grab the entire plane (it can be grabbed on any white spot) and shift it around the window.



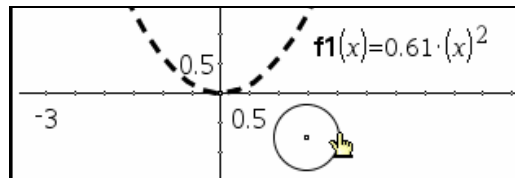
For certain classes of expressions one can grab the graph, change its shape, and observe how the function term changes accordingly.



Using the **Attribute** command, one can change the appearance of a graph or how the function graph is displayed.



TI-Nspire also offers many of the dynamic geometry features known from Cabri Geom  tre such as drawing a circle:



Lists & Spreadsheet

We use the **Insert>Lists&Spreadsheet** command to add a page connected to a spreadsheet. This spreadsheet offers both column logic and cell logic. One can do the standards of what one can do with standard numerical spreadsheets; in addition this spreadsheet can carry and process algebraic expressions. The only limitation may be that within the cells the expressions are displayed in linear format, only by selecting a cell one can view its contents in two-dimensional format at the bottom of the page window.

	A	t	B	d	C	i	D	E
•			=d(t,x)		=∫(t,x)			
1	x^2		2*x		x^3/3			
2	x^4		4*x^3		x^5/5			
3	x^8		8*x^7		x^9/9			
4	x^15		15*x^14		x^16/16			
5	x^20		20*x^19		x^21/21			
6								
7								
A1 x^2								

This is ideal for having students recognize patterns. Once they have an idea, such as for the above, the teacher may challenge them by adding an example for which the validity of their pattern needs some additional thinking, such as in the following screen image.

5	x^20	20*x^19	x^21/21		
6	x^(-3)	-3/x^4	-1/(2*x^2)		

Notes

We use the **Insert>Notes** command to add a page for producing text.

This text processing application offers standard features such as **bold**, underlined, sub^{scripted}, ^{super}scripted, etc.

It allows for many special characters such as for denoting angles, $\angle(ABC)$, triangles, $\Delta(ABC)$, segments, \overline{AB} , and more.

If one refers to definitions made in other applications, such as $f1(x)$, these can be selected and evaluated: $f1(x)=f1(x)$

By applying the **Evaluate selection** command, the selected expression is algebraically simplified, i.e. also the text processing application is connected with the computer algebra application.

If one refers to definitions made in other applications, such as $f1(x)$, these can be selected and evaluated: $f1(x)=x^{(2)}$

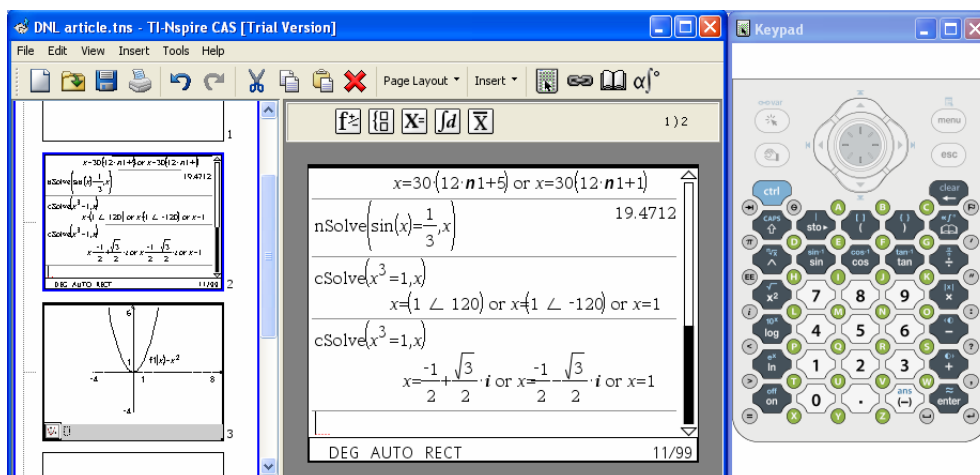
Miscellaneous

Within a TI-Nspire document, user-defined functions and variable assignments are local to a “Problem”, within which they are valid in all pages. Other problems in the same document have their own local variables.

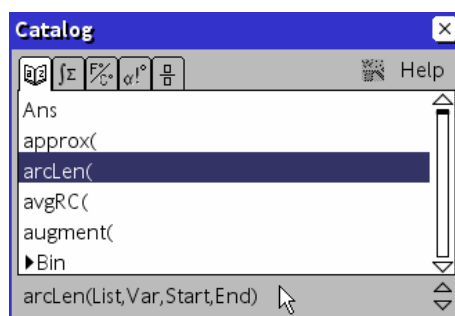
A very useful command is the general **Undo** command, which undoes whatever has been done last. A corresponding **Redo** command is also available.



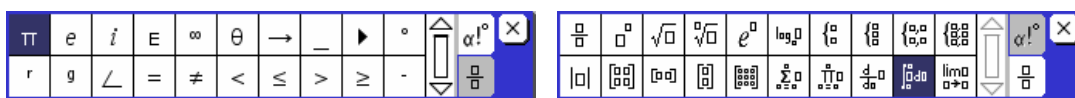
For fast access to certain symbols or build-in functions, one can switch on the **Keypad**, which can be put next to the application window for convenient input. This is how the calculator keyboard looks like!



Other sources for supporting the inputting of expressions are the **Catalog** with a list of all build-in functions in alphabetical order including a quick “online help” in form of “expressive” templates at the bottom of the window, ...



... and a **Symbol Palette** for special symbols and input templates.



Closing Remark

TI-Nspire's innovative double-platform design (handheld + 100 % compatible computer software) and the seamless integration of the various components (CAS, graphing, geometry, spreadsheet, text processing) make it a very powerful teaching and learning aid for teachers and students that is unique on the market and has great potential. Please help testing it. I'll be happy to collect the feedback and forward it to the appropriate people.

Beyond Polynomial Regression and Beyond quartreg on the TIs!

Josef Böhm, Würmla, Austria

Well known – since several years even in school mathematics – are linear and quadratic regression, both based on the least squares method. One of the reasons that regression models found their way in class room is the increasing importance of statistics and another one is the availability of tools which make the tiresome numerical calculation easier or even unnecessary.

Given is a set of bivariate data $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$. Presentation of the data in a scatter diagram sometimes leads to the conjecture that there might be a linear relationship between x - and y -values. Regression tries to find a linear model function $f(x) = kx + d$ such that the sum of the squares of the differences between real and theoretical function values becomes minimal:

$$\Omega(k, d) = \sum_{i=1}^n (f(x_i) - y_i)^2 = \sum_{i=1}^n (kx_i + d - y_i)^2 = \text{Minimum}$$

This is an extremal value problem with two variables k and d . The values of them can be found by partial differentiation of $\Omega(k, d)$ with respect to k and d and then solving the resulting system of linear equations for k and d .

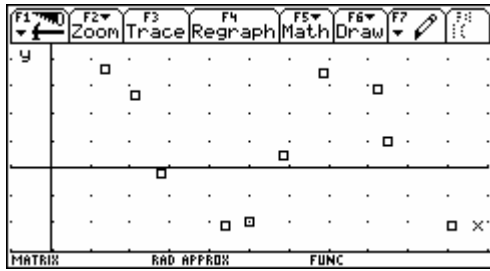
This method is going to be generalized. Let's have a demonstration example:

(I will start working on the handheld, the DERIVE realisation will follow.)

x_i	1,36	2,14	2,76	4,40	4,99	5,85	6,89	8,25	8,50	10,09
y_i	3,65	2,65	-0,20	-2,12	-1,96	0,49	3,44	2,88	0,97	-2,11

Store the data in two lists $x1$ and $y1$ and have a first impression by inspecting the scatter diagram:

We can be sure that linear regression is not appropriate in this case. We could try polynomial function of higher degree, but the scatter diagram reminds us on trig functions, doesn't it?



So we look for an approximating function $\Phi(x)$ defined as a linear combination of the set (list) of functions:

$$\{1, \sin(x), \cos(x)\}.$$

$$\Phi(x) = a \cdot 1 + b \cdot \sin(x) + c \cdot \cos(x)$$

We obtain an extremal value problem with three variables a , b and c :

$$\Omega(a, b, c) = \sum_{i=1}^N (\underbrace{a_1 f_1(x_i) + a_2 f_2(x_i) + a_3 f_3(x_i)}_{a \cdot 1 + b \cdot \sin(x_i) + c \cdot \cos(x_i)} - y_i)^2 = \text{Minimum}$$

$$\frac{\partial \Omega}{\partial a} = \sum_{i=1}^N (2(a + b \sin(x_i) + c \cos(x_i) - y_i) \cdot 1) = 0$$

$$\frac{\partial \Omega}{\partial b} = \sum_{i=1}^N (2(a + b \sin(x_i) + c \cos(x_i) - y_i) \cdot \sin(x_i)) = 0$$

$$\frac{\partial \Omega}{\partial c} = \sum_{i=1}^N (2(a + b \sin(x_i) + c \cos(x_i) - y_i) \cdot \cos(x_i)) = 0$$

The resulting equations can be cancelled by 2, expanded and rearranged:

$$\begin{aligned} a \sum_N 1 \cdot 1 + b \sum_N \sin x_i \cdot 1 + c \sum_N \cos x_i \cdot 1 &= \sum_N y_i \cdot 1 \\ a \sum_N 1 \cdot \sin x_i + b \sum_N \sin x_i \cdot \sin x_i + c \sum_N \cos x_i \cdot \sin x_i &= \sum_N y_i \cdot \sin x_i \\ a \sum_N 1 \cdot \cos x_i + b \sum_N \sin x_i \cdot \cos x_i + c \sum_N \cos x_i \cdot \cos x_i &= \sum_N y_i \cdot \cos x_i \end{aligned}$$

The system is linear again. Our task is to set up the system for the given data ($N = 10$) and solve it. Then add the graph of the “regression line” to the scatter plot. We can use the lists $x1$ and $y1$.

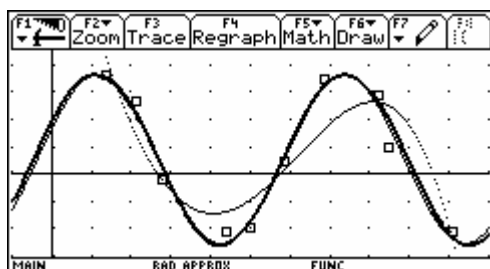
Note: $\text{sum}(\text{list})$ gives the sum of all elements of a list.

$$10a + 1.533626b - 1.336507c = 7.690000$$

$$1.533626a + 6.006008b - 0.829608c = 16.119240$$

$$-1.336507a - 0.829608b + 3.993992c = 2.871270$$

which leads to: $\Phi(x) = 0.544245 + 2.748197 \sin x + 1.471855 \cos x$.



The Calc-Menu ‡ in the DATA-MATRIX-Editor of the TI offers option SinReg which gives a sinusoidal regression line.

The result is:

$$y(x) = 3.09219 \sin(1.02073x + 0.37917) + 0.54421$$

thick: $\Phi(x)$, line: $y(x)$ and dot: quartreg

F1	F2	F3	F4	F5	F6
Algebra	Calc	Other	PrgmIO	Clean Up	
$\begin{bmatrix} -1.336507 & -.829608 & 3.993992 \\ 1.000000 & 0.000000 & 0.000000 & .544245 \\ 0.000000 & 1.000000 & 0.000000 & 2.748187 \\ 0.000000 & 0.000000 & 1.000000 & 1.471855 \end{bmatrix}$					
sum((y6(x1) - y1)^2)					1.772076
sum((y7(x1) - y1)^2)					1.644764
sum((y7(x1) - y1)^2)					
MATRIX RAD APPROX FUNC 5/30					

We compare the quality of both regression lines by calculating the sum of the squares of the residuals. The TI-built-in trig regression proves to be a little bit better.

Calculating the trig regression line is much much more complicated (see DNL# 46)

The method shown above can be performed very elegant by using matrix algebra. First of all we create matrix M , the so called **Normalmatrix**:

$$M = \begin{pmatrix} f_1(x_1) & f_1(x_2) & f_1(x_3) & \dots & f_1(x_{10}) \\ f_2(x_1) & f_2(x_2) & f_2(x_3) & \dots & f_2(x_{10}) \\ f_3(x_1) & f_3(x_2) & f_3(x_3) & \dots & f_3(x_{10}) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ \sin(x_1) & \sin(x_2) & \sin(x_3) & \dots & \sin(x_{10}) \\ \cos(x_1) & \cos(x_2) & \cos(x_3) & \dots & \cos(x_{10}) \end{pmatrix}$$

What is the result of $M \cdot M^T$?

Solving the simultaneous equation is easily done:

Please compare with the solution from above!

F1	F2	F3	F4	F5	F6
Algebra	Calc	Other	PrgmIO	Clean Up	
sum(y1*cos(x1))					2.871270
[normm*normm^T]^-1					$\begin{bmatrix} 7.69 \\ 16.119239811774 \\ 2.8712700538912 \end{bmatrix}$
					.544245
					2.748187
					1.471855
MATRIX RAD APPROX FUNC 17/30					

Here is a problem for you (students):

x_i	-0,51	1,34	3,57	5,77	7,34	9,49	12,37	14,89
y_i	-3,15	-12,90	-9,14	-13,54	-8,81	-10,33	-5,20	-3,23
x_i	15,34	18,09	19,86	21,25	24,48	26,04	28,42	30,83
y_i	2,61	9,17	11,49	16,02	26,80	31,01	38,98	46,21

The approximating function shall be a linear combination of:

$$f_1(x) = 1, f_2(x) = x^2, f_3(x) = (x+1)^{-1} \text{ und } f_4(x) = (x+2)^{-2}.$$

Try to use appropriate TI-functions to keep editing as little as possible.

You can read off the solution:

F1	F2	F3	F4	F5	F6
Algebra	Calc	Other	PrgmIO	Clean Up	
					-15.948999
					.066795
					27.225414
					-94.796100
$-94.7961 \over (x+2)^2 + 27.2254 \over x+1 + .066795 \cdot x^2 - 15.95$					Done
$\dots x+1)+0.066795x^2-15.95 \rightarrow y1(x)$					
MATRIX RAD APPROX FUNC 28/30					

Possible questions for students:

Can we use a polynomial of higher degree as regression line? Do you find another sensible set of basic functions? Compare the quality of the regression models by calculating the SSE (sum of squared errors = sum of squares of residuals). This method can help to overcome the restriction of polynomial regressions on the TI. Do you have any idea how to do?

Realisation with a DERIVE-function

Transfer to DERIVE is very easy, because we can use the TABLE-function very efficiently to create the normalmatrix. A three-lines program does the whole job:

```

genreg(flist, pts, nm, rs) :=
  Prog
#1:   nm := REST(TABLE(flist, x, pts_111))
      rs := [VECTOR(pts_112·nmik, k, DIM(flist))]
      ((nm·nm')^(-1)·rs)_111·flist

```

```

#2:   test :=
      [ 1.36  3.65
        2.14  2.65
        2.76 -0.2
         4.4 -2.12
        4.99 -1.96
        5.85  0.49
        6.89  3.44
        8.25  2.88
         8.5  0.97
       10.09 -2.11 ]

```

```
#3:   genreg([1, SIN(x), COS(x)], test)
```

```
#4:   1.471854558·COS(x) + 2.748187037·SIN(x) + 0.5442452255
```

Big surprise? Or not?

```
FIT([x, a + b·SIN(x) + c·COS(x)], test)
```

```
1.47185·COS(x) + 2.74818·SIN(x) + 0.544245
```

Please read Karsten Schmidt's comment on this kind of regression (page 5) and read the paragraph on the FIT-function in the Online Help of DERIVE.

You might ask if our work was superfluous and waste of time? My opinion is: No, because we could make a "Black Box" to a "White Box" with relatively little effort.

Let's compare with the sinreg(datamatrix) program for trigonometric regression.

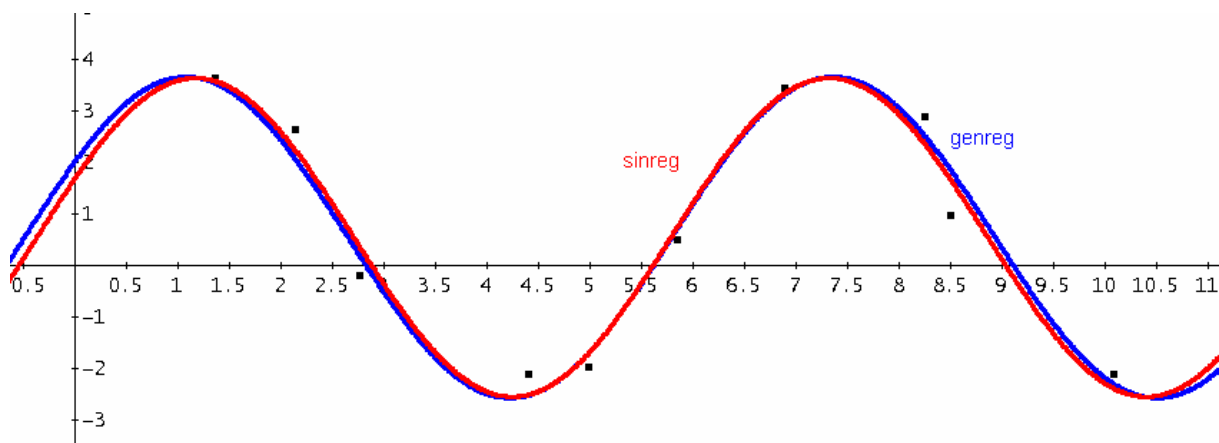
In 2002 Don Phillips provided a wonderful tool based on the Levenberg-Marquardt and Gauss-Newton Methods for nonlinear regression models.³

```
#5:   sinreg(test)
```

```
#6:   [ Regressionslinie: y = 0.544210 - 3.09218·SIN(1.02073·x + 3.52075)
      SummeRes^2:                1.64476 ]
```

```
#7:   TABLE(1.47185·COS(x) + 2.74818·SIN(x) + 0.544245, x, -1, 12.5, 0.01)
```

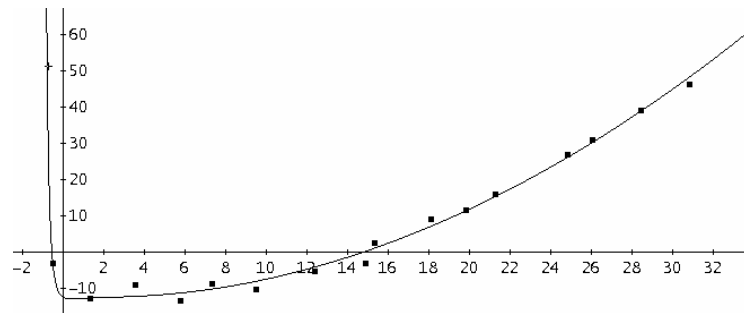
```
#8:   TABLE(0.54421 - 3.09218·SIN(1.02073·x + 3.52075), x, -1, 12.5, 0.01)
```



³ Derive Newsletter #46, *Statistics-Tools for DERIVE and the TI-92*

test2 is the data matrix of the 2nd example (and try again with the FIT-function and compare!):

$$\begin{aligned} \#6: & \text{EXPAND} \left(\text{genreg} \left(\left[1, x^2, \frac{1}{x+1}, \frac{1}{(x+2)^2} \right], \text{test2} \right) \right) \\ \#7: & -\frac{94.79609995}{(x+2)^2} + \frac{27.22541355}{x+1} + 0.06679516555 \cdot x^2 - 15.94899938 \end{aligned}$$



Programming for the CAS-TI-Calculator

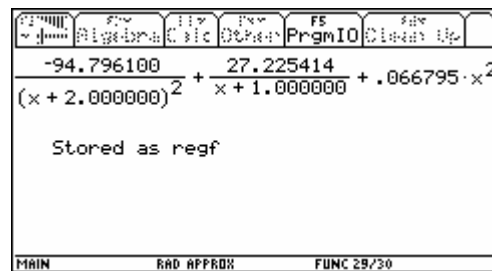
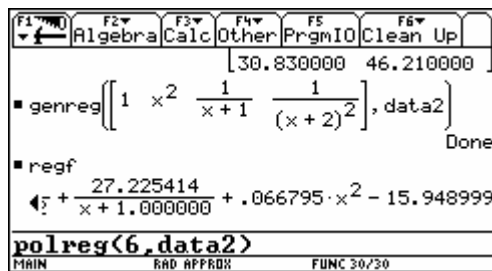
genreg(functionlist, data matrix) is the respective program for the TIIs.

A special list of functions (powers of x) leads immediately to a higher degree polynomial regression $\text{polreg}(\text{degree}, \text{data matrix})$. (See the screen shot on the next page.)

```
genreg
(flist,pts)
Prgm
  Local nm,rs,xv,yv
  ptsT[1]→xv:ptsT[2]→yv
  newMat(dim(flist)[2],dim(xv)[2])→nm
  newMat(dim(flist)[2],1)→rs
  For k,1,dim(flist)[2]
    For l,1,dim(xv)[2]
      limit(flist[1,k],x,xv[1,l])→nm[k,l]
    EndFor
    dotP(yv,nm[k])→rs[k,1]
  EndFor
  (nm*nmT)(-1)*rs→rs
  dotP(rsT[1],flist[1])→regf
  Disp regf
  Disp ""
  Disp "    Gespeichert unter regf"
EndPrgm
```

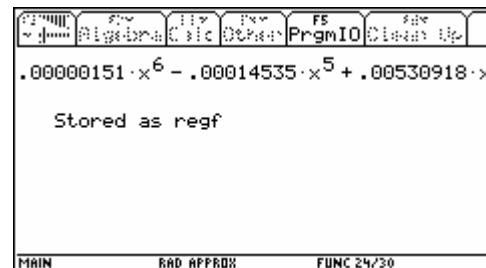
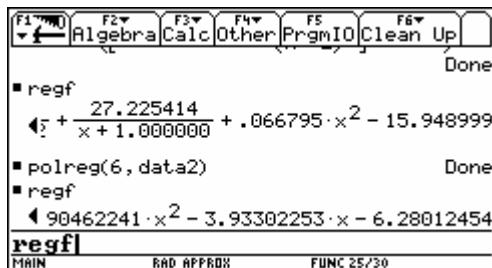
F1	F2	F3	F4	F5	F6
Algebra	Calc	Other	PrgmIO	Clean Up	
1.36	3.65		1.3600	3.6500	
2.14	2.65		2.1400	2.6500	
2.76	-.2		2.7600	-.2000	
4.4	-2.12		4.4000	-2.1200	
4.99	-1.96		4.9900	-1.9600	
5.85	.49	→ data	5.8500	.4900	
6.89	3.44		6.8900	3.4400	
nreg([1,sin(x),cos(x)],data)					

F1	F2	F3	F4	F5	F6
Algebra	Calc	Other	PrgmIO	Clean Up	
1.471855·cos(x) + 2.748187·sin(x) + .544245					
Stored as regf					
MAIN RAD APPROX FUNC 30/30					



Compare the results!

The TI-program for polynomial regression `polreg()` refers to `genreg()`:



Compare again with the DERIVE result applying the FIT-function:

#9: $\text{FIT}\left(\begin{bmatrix} 6 & 5 & 4 & 3 & 2 \\ x, a \cdot x^6 + b \cdot x^5 + c \cdot x^4 + d \cdot x^3 + e \cdot x^2 + f \cdot x + g \end{bmatrix}, \text{test2}\right)$

#10: $0.00000150806 \cdot x^6 - 0.000145352 \cdot x^5 + 0.00530917 \cdot x^4 - 0.0931230 \cdot x^3 + 0.904622 \cdot x^2 - 3.93302 \cdot x - 6.28012$

A Closing Example:

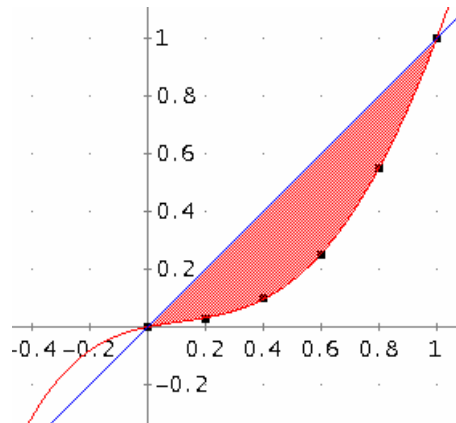
A Lorenz Curve is defined by the points (0|0), (0.2|0.03), (0.4|0.1), (0.6|0.25), (0.8|0.55) und (1|1) (= Quintil-values). The curve shall be modelled by a quartic passing the origin.

This is no problem in DERIVE:

#17: $\text{lrenz} := \begin{bmatrix} 0 & 0 \\ 0.2 & 0.03 \\ 0.4 & 0.1 \\ 0.6 & 0.25 \\ 0.8 & 0.55 \\ 1 & 1 \end{bmatrix}$

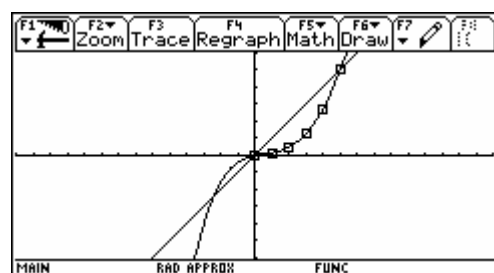
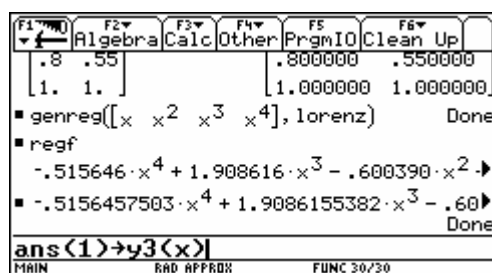
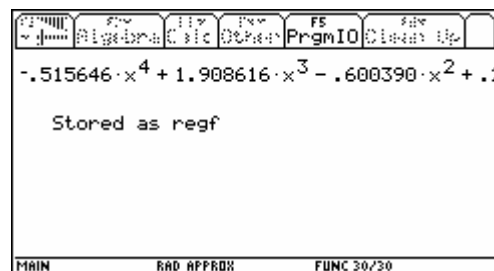
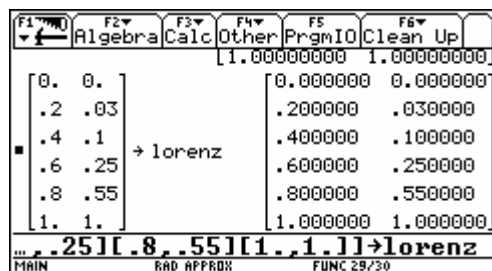
#18: $\text{FIT}\left(\begin{bmatrix} 4 & 3 & 2 \\ x, a \cdot x^4 + b \cdot x^3 + c \cdot x^2 + d \cdot x \end{bmatrix}, \text{lrenz}\right)$

#19: $-0.5156457503 \cdot x^4 + 1.908615537 \cdot x^3 - 0.6003901062 \cdot x^2 + 0.2078187250 \cdot x$



- Note: In economy the Lorenz curve is used to represent the inequality of eg. income distribution among different groups in the population of a country. This inequality of the distribution is measured by the GINI coefficient, which measures how far the Lorenz curve falls below $y = x$. It is the ratio of the area between $y = x$ and the Lorenz curve and the area between $y = x$ and $y = 0$. (This is a very welcome application of the defined integral.)

From today on this is no problem for you as a TI-User, too:



For higher degree polynomial regression it can happen that the matrix will become singular caused by internal calculation inaccuracy.

References:

- [1] R. J. Hill & T. A. Keagy, *Elementary Linear Algebra with DERIVE*, Chartwell-Bratt 1995
- [2] H. D. Hinkelmann u.a., *Mathe mit Gewinn 2 & 3*, oebvhpt 2005, 2006
- [3] H. D. Hinkelmann u.a., *Mathe mit Gewinn 3*, oebvhpt, 2006
- [4] Josef Böhm, *Matrizenrechnung mit dem TI-92*, T³-Skriptum (www.acdca.ac.at)
- [5] Ronald Harshbarger & J J Reynolds, *Mathematical Applications*, Houghton Mifflin 2000
- [6] Karsten Schmidt & Götz Trenkler, *Einführung in die Moderne Matrix-Algebra*, Springer 2006
- [7] *Derive Newsletter #46*, March 2002

Contour Plots – Level Curves

Peter Schofield, UK & Josef Böhm, Austria

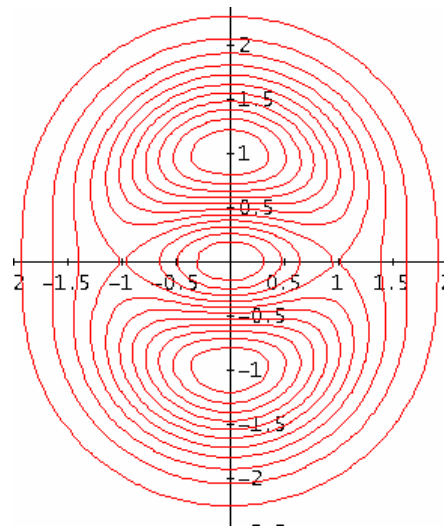
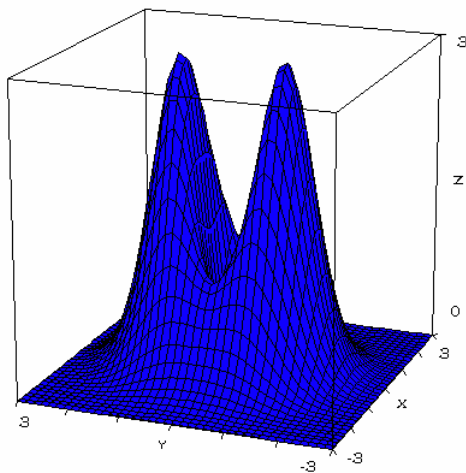
In DNL#59 appeared a request from Manuel Garcia Mateos for plotting level curves (= contour plots) with DERIVE. Manuel was right complaining that other major CAS-programs (MAPLE, MATHEMATICA) provide packages for creating contour plots either in the xy -plane or as a 3D-plot. Producing a plot in the xy -plane is not so difficult using the VECTOR-command and DERIVE's ability to create implicit plots. As a "Graphic Freak" I wanted to overcome the deficiency of presenting the contour curves in 3D and started thinking about a program. Now, since I am able to use three thicknesses of lines in the plane we have more varieties plotting even in plane. Peter Scofield who is also a graphics enthusiast and a real graphics wizard joined the party and gave his version of facing the challenge. Moreover Peter produced a toolbox for plotting implicit forms of three variables and found an improved version of NSOLUTIONS and NSOLVE.

I'll demonstrate what we did using function f:

Finding the contour plots in xy -plane is not too difficult:

$$\#1: f := (x^2 + 3 \cdot y^2) \cdot e^{1-x-y^2}$$

$$\#2: \text{VECTOR}(f = z, z, 0, 3, 0.25)$$



Two more examples of plane contour plots with their graphs on the next page.

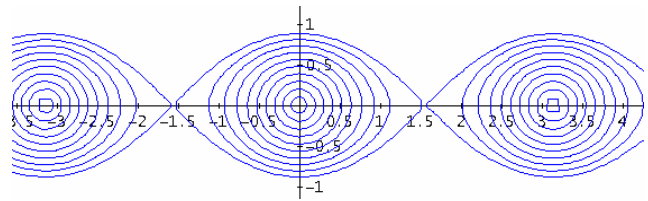
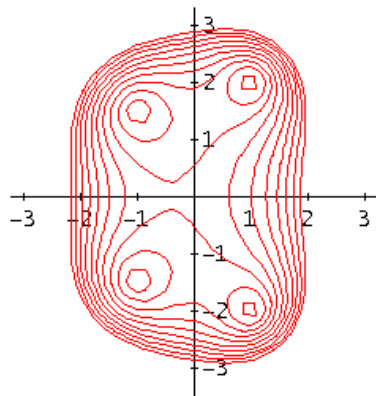
$$\#15: f2 := \sqrt{(16 \cdot x^8 + 8 \cdot x^6 \cdot (8 \cdot y^2 + 17) + 112 \cdot x^5 + x^4 \cdot (96 \cdot y^4 + 136 \cdot y^2 + 801) + 28 \cdot x^3 \cdot (17 - 8 \cdot y^2) + 2 \cdot x \cdot (32 \cdot y^6 - 68 \cdot y^4 - 1247 \cdot y^2 + 1186) - 28 \cdot x \cdot (12 \cdot y^4 - 17 \cdot y^2 - 64) + 16 \cdot y^8 - 136 \cdot y^6 + 801 \cdot y^4 - 1980 \cdot y^2 + 4096)}$$

$$\#16: \text{VECTOR}(f2 = z, z, 0, 200, 20)$$

$$\#23: f3 := \frac{e^{-y} \cdot \sqrt{(e^{4 \cdot y} + 2 \cdot e^{2 \cdot y} \cdot (1 - 2 \cdot \cos(x)^2) + 1)}}{2}$$

$$\#24: \text{VECTOR}(f3 = z, z, 0, 1, 0.1)$$

We want to see the level curves in 3D-plot – either on the surface or as projection on the xy -plane – and we would like to plot the 2D-level curves in various thicknesses.



For the following I need my auxiliary function `mysolutions`, which – as I am hoping – finds all numeric solutions of equation `equ` vor variable `equ_v` in the range between $x = \text{start}$ and $x = \text{end}$.

```
#3:  mysolutions(equ, equ_v, start, end, step := 0.1) := APPEND(VECTOR(NSOLUTIONS(equ, equ_v, a_, a_ + step),
    a_, start, end - step, step))
```

This is the first half part of my programm `contour()`.

```
contour(f, level, xst, xend, yst, yend, xstep := 0.2, ystep := 0.2, acc := 0.2, equ, x_, y_, pts) :=
  Prog
  pts := []
  x_ := xst
  f := SUBST(LHS(f) - RHS(f), z, level)
  Loop
    equ := SUBST(f, x, x_)
    If ~ equ
      pts := APPEND(pts, [[]])
      pts := APPEND(pts, [[]])
      pts := APPEND(pts, VECTOR([[x_, v, level]], v, mysolutions(equ, y, yst, yend, acc)))
#8:  x_ := x_ + xstep
    If x_ > xend exit
  y_ := yst
```

I enter any equation with three variables x, y and z as `f`. `level` is the distance of the level plane from the xy -plane. The plot range is defined by $xst \leq x \leq xend$ and $yst \leq y \leq yend$. In the 3rd line `f` will change to an implicit form $f(x,y) = \text{level}$. In the following loop we substitute x by all values from `xst` to `xend` with an increment of `xstep` (default value 0.2) and try to find all solutions for y , which leads to points on the surface – if there are solutions. The first two parts in the IF-construct are necessary for special cases (`f` from line 3 can be returned as `false`, ...). If you are fine with the default values, then entering parameters ends with `yend`. `acc` sets the accuracy for the `mysolutions`-procedure.

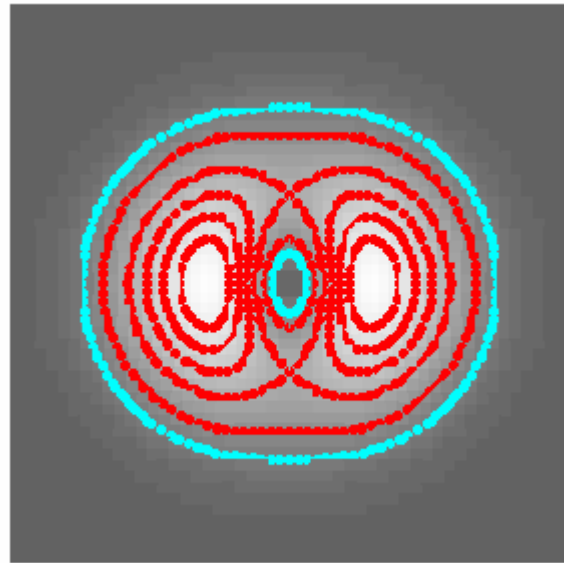
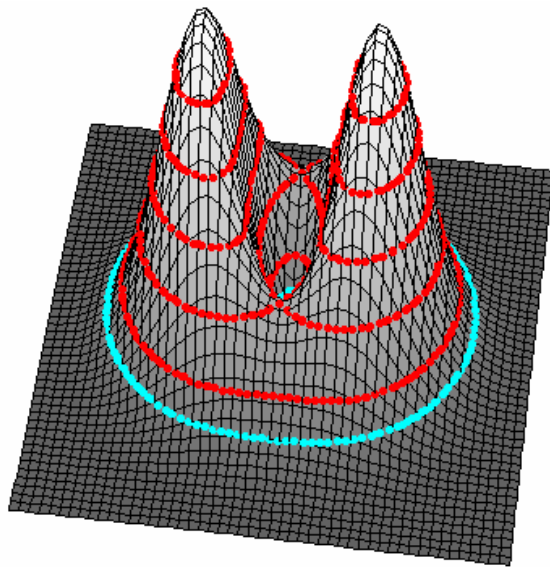
Then we repeat the same procedure for the sequence of y -values to have a tighter point set.

See my first results (don't mind some calculation time!)

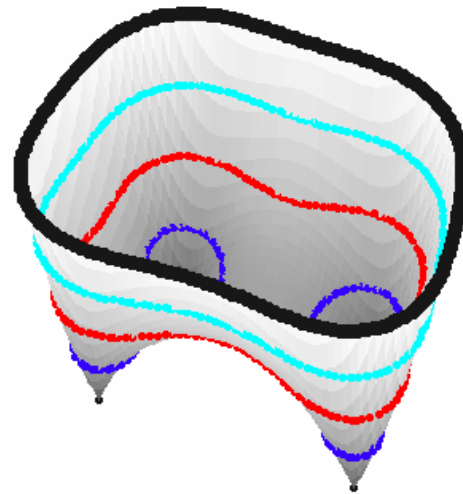
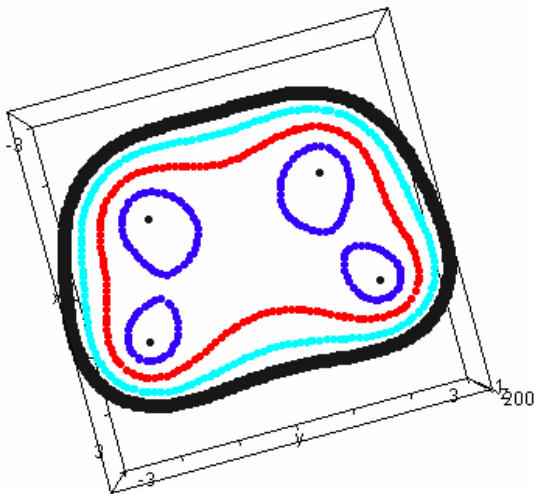
```
#9:  contour(f = z, 0.25, -3, 3, -3, 3, 0.1, 0.1, 0.5)
```

```
#10: VECTOR(contour(f = z, z, -3, 3, -3, 3, 0.1, 0.1, 0.5), z, 0.5, 2.5, 0.5)
```

Highlight the expressions, switch to the 3D-Plot Window (set Options > Approximate before Plotting) and Insert > Plot. When DERIVE is ready (in the background) you will be asked for the Point Size (Small / Medium / Large). Try any of them. Then you will be asked for the Plot Colour. Set Scheme Custom and set Top Maximum and Minimum the same colors.



Contour plots of #15 from above:



This is not so bad. But if you choose Point Size Small, the points are not really small. I wanted to have only dots as points. It is not possible to have discrete points appearing as dots – either the points are connected or if discrete, then small (which is not really small), medium or large. (This is what I know). So I wanted to have a “contour dust”. Peter gave the idea: plot a segment of length 0. [1,2,3;1,2,3] shows a tiny dot. So I only had to change the output form of contour to get `contourdot()` with the same parameters as before.

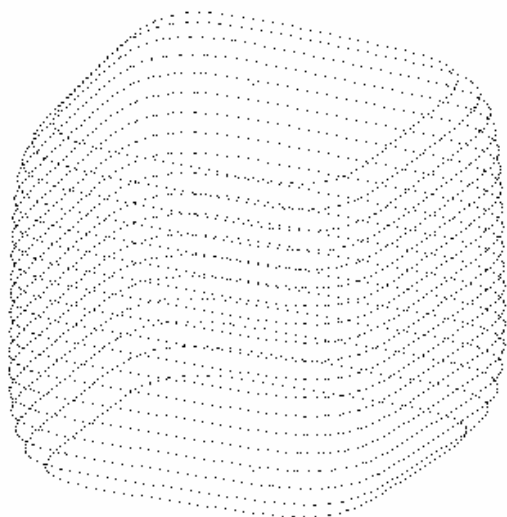
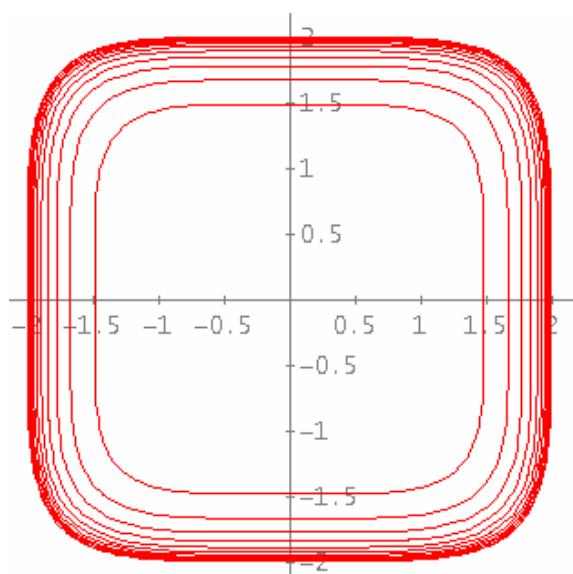
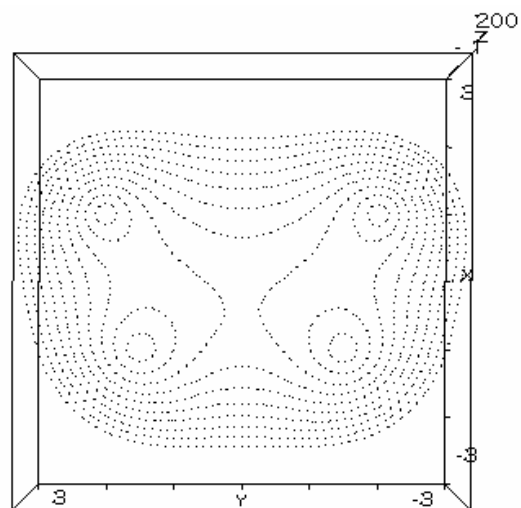
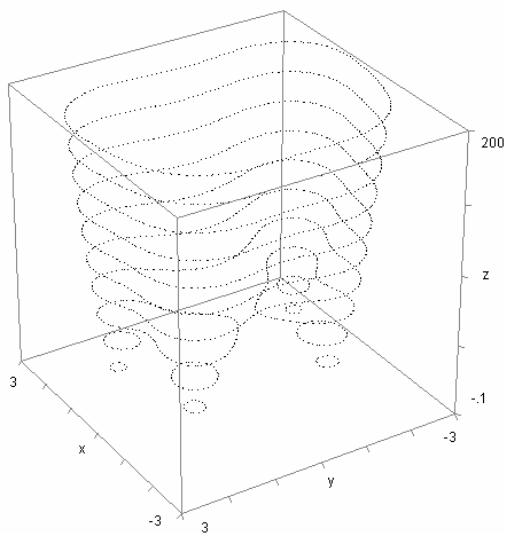
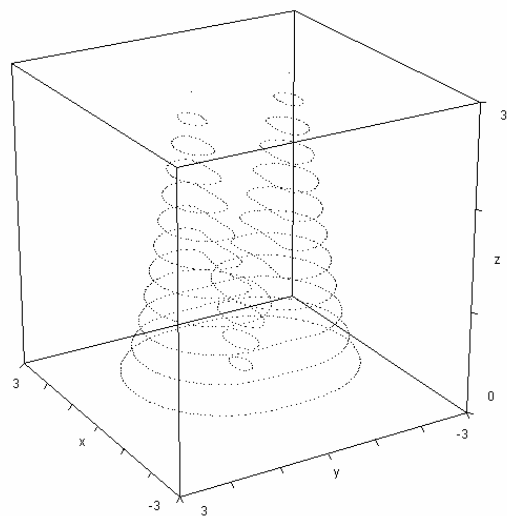
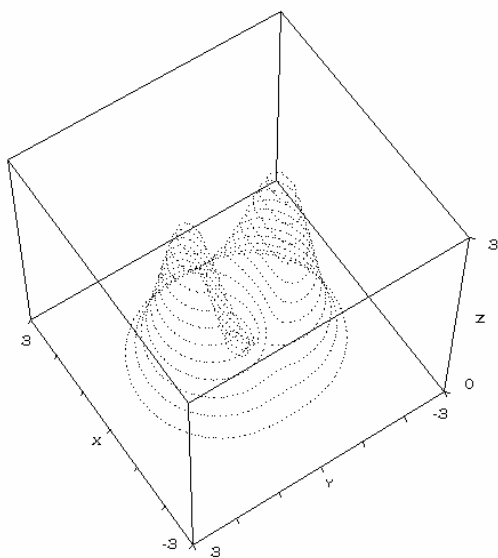
```
#12: VECTOR(contourdot(f = z, z, -3, 3, -3, 3, 0.1, 0.1, 0.5), z, 0.25, 3, 0.25)
```

```
#22: VECTOR(contourdot(f2 = z, z, -3, 3, -3, 3, 0.1, 0.1, 0.5), z, 0, 200, 20)
```

```
#36: VECTOR(|x|^5 + |y|^5 + |z|^5 = 32, z, 0, 2, 0.1)
```

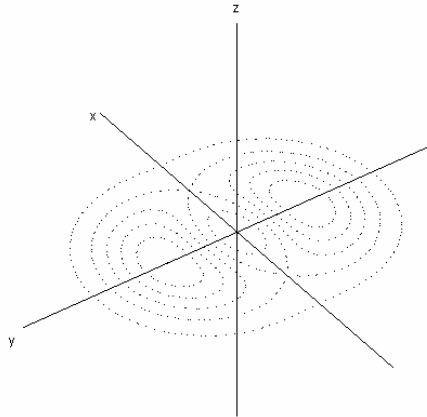
```
#37: VECTOR(contourdot(|x|^5 + |y|^5 + |z|^5 = 32, z, -2, 2, -2, 2, 0.1, 0.1, 0.5), z, -2, 2, 0.2)
```

See the plots on the next page:



And finally there is `contourdotplane()` which gives the projections of the level curves in the xy -plane but as a 3D-object:

```
#14: VECTOR(contourdotplane(f = z, z, -3, 3, -3, 3, 0.1, 0.1, 0.5), z, 0.5, 3, 0.5)
```



As it was mentioned in the contribution about plotting thick lines, it was not too difficult to adapt the basic idea creating contour plots for doing implicit plots with thick lines (which are in fact only thick points in a row).

`impplot()` is a adaptation of `contourdotplane()`.

Peter sent a mail together with his files `polycontour.mth` and `polycontour.dfw` and just recently an improved version:

```
#1: Procedures (based on Josef's contour idea) for 3D-plotting contours of surfaces of form f(x,y,z) (=0)
    or f(x,y,z) = g(x,y,z) using a relatively quick N_SOLUTIONS procedure.
```

```
N_SOLUTIONS(f, x, a, b, c, i, z, p := []) :=
  Prog
```

Peter found a quite better auxiliary program for finding the numeric solution of an equation. I'll present this programs `N_SOLUTIONS()` and `N_SOLVE()` in an extra paragraph.

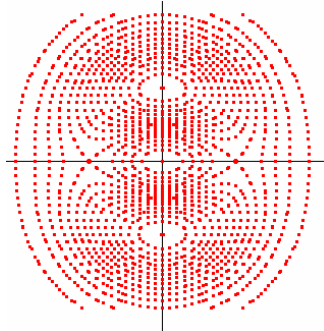
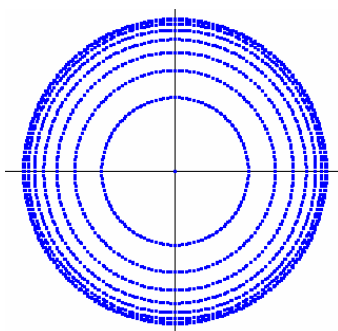
```
#3: 2D-CONTOUR-POINT-PLOTS
```

```
ContourPts_2D(f, z_, stx := -4, endx := 4, sty := -4, endy := 4, stepx := 0.1, stepy := 0.1, s, f_, p := []) :=
  Prog
```

Approximate and 2D-Plot

```
#5: VECTOR(ContourPts_2D(x2 + y2 + z2 - 16, 1), 1, 0, 4, 0.5)
```

```
#6: VECTOR(ContourPts_2D((x2 + 3*y2)*EXP(1 - x2 - y2) - z, z_, -2, 2, -2, 2), z_, 0, 3.2, 0.2)
```



For 3D-representations Peter offers ContourPts_XY() and ContourDots_XY(). Using the first program you can choose between the three point sizes Small / Medium / Large and the second one gives us the “dust”. An important EXTRA is the fact that his function can process Boolean expressions and operators.

#8: 3D-CONTOUR-POINT-PLOTS

```
ContourPts_XY(f, z_, stx := -5, endx := 5, sty := -5, endy := 5, stepx := 0.1, stepy := 0.1, s, f_, p := []) :=
  Prog
```

```
#10: ContourDots_XY(f, z_, stx := -5, endx := 5, sty := -5, endy := 5, stepx := 0.1, stepy := 0.1) := ADJOIN([],
  VECTOR([u, u], u, REST(ContourPts_XY(f, z_, stx, endx, sty, endy, stepx, stepy))))
```

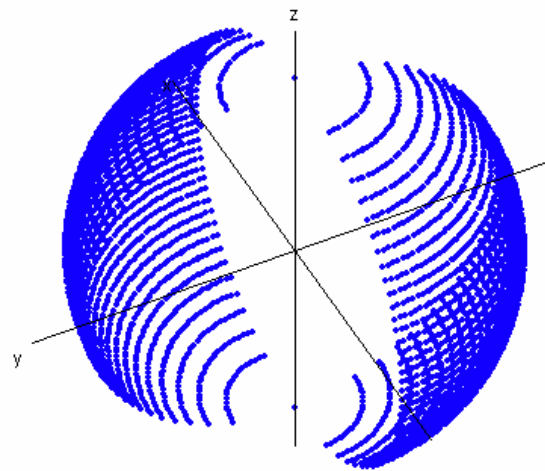
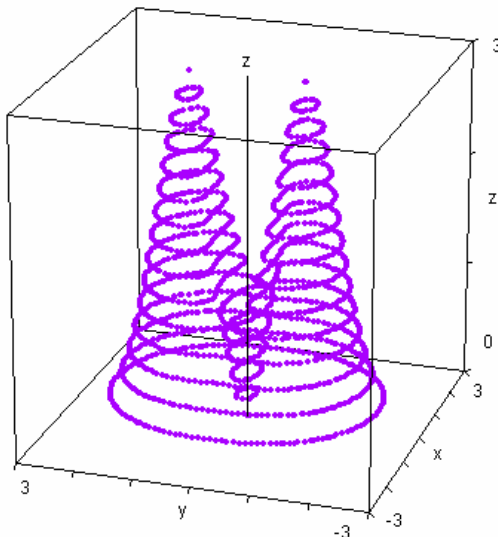
```
#11: [Precision := Approximate, PrecisionDigits := 4, Notation := Decimal]
```

#12: Approx. and 3D-PLOT:

```
#13: VECTOR(ContourPts_XY(z = (x^2 + 3*y^2) * EXP(1 - x^2 - y^2), z_, -3, 3, -3, 3), z_, 0, 3.2, 0.2)
```

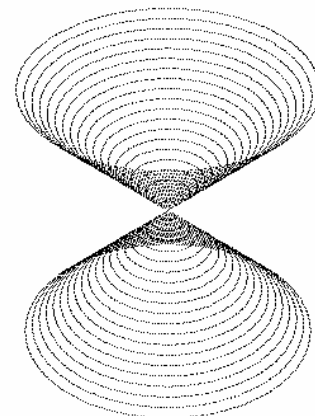
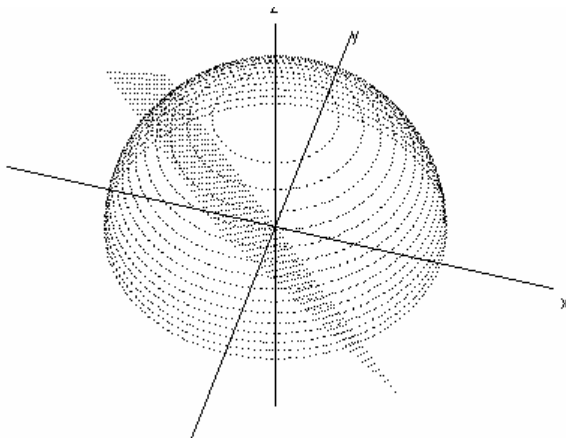
#14: It can also process Boolean expressions and operators!! Approx. and 3D-PLOT:

```
#15: VECTOR(ContourPts_XY(x^2 + y^2 + z^2 = 16 & x*y ≥ 0, z_, -4, 4, -4, 4), z_, -4, 4, 0.2)
```



```
#16: VECTOR(ContourDots_XY(x^2 + y^2 + z^2 = 9 & x + y + z = 0, z_, -3, 3, -3, 3), z_, 0, 3, 0.2)
```

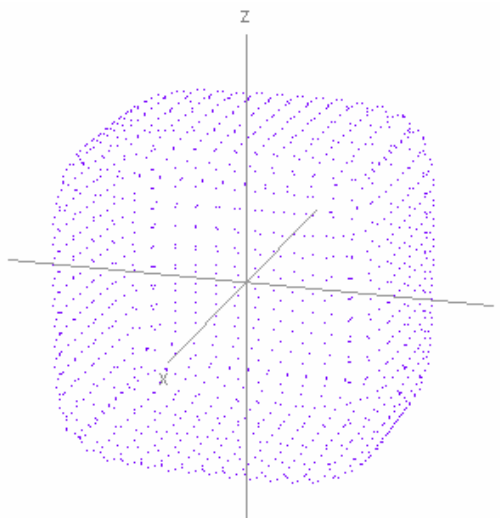
```
#17: VECTOR(ContourDots_XY(x^2 + y^2 - z^2, 1), 1, -5, 5, 0.25)
```



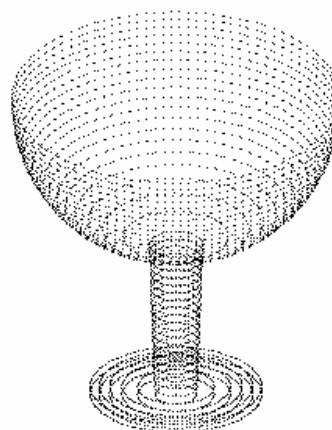
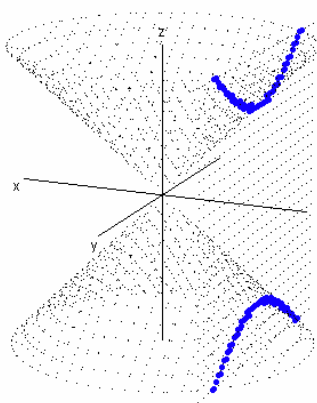
In my short introduction I announced that Peter provided a file for doing implicit 2D and 3D-plots. Here are some mouth watering examples. I'll come back to that in the next newsletter.

Among the files which you can download you will find contours_j.dfw, contours_j.mth, polycontours.dfw and polycontours.mth. The dfw-files contain the worked examples and the mth-files are containing the functions only. They should be loaded as utility files in the background.

Happy scanning,
Peter & Josef



#66: `ImplicitDots(|x|^5 + |y|^5 + |z|^5 - 2, -2, 2)`



mysolutions versus N_SOLUTIONS

Josef's version for numeric solutions of equation equ (first lines of code):

```
mysolutions(equ, equ_v, start, end, step := 0.1, list, sol_list) :=
  Prog
    list := APPEND(VECTOR(NSOLUTIONS(equ, equ_v, a_, a_ + step), a_, start, end - step, step))
    If DIM(list) = 0
      RETURN list
    sol_list := [FIRST(list)]
#1: Loop
```

Peter's version for numeric solutions of equation f (first lines of code):

```
N_SOLUTIONS(f, x, a, b, c, i, z, p := []) :=
  Prog
    c := [[a, b]]
    Loop
      i := DIM(c)
      Loop
        a := (FIRST(c))_1
        b := (FIRST(c))_2
        c := REST(c)
        If a < b
#2: Prog
      z := NSOLUTIONS(f, x, a, b)
```

There is also a N_SOLVE-function which might substitute the DERIVE NSOLVE. Inspect the file.

See first, what DERIVE is offering::

$$\#3: \text{NSOLUTIONS}\left(5 \cdot e^{-0.2 \cdot x} \cdot \sin(5 \cdot x) = \frac{1}{x}, x\right)$$

$$\#4: [-5.65258199]$$

$$\#5: \text{SOLUTIONS}\left(5 \cdot e^{-0.2 \cdot x} \cdot \sin(5 \cdot x) = \frac{1}{x}, x\right)$$

$$\#6: [-9.424133421, \infty, -\infty, \infty, \infty + 47.1238898 \cdot i, \infty + 15.70796326 \cdot i, \infty - 15.70796326 \cdot i, \infty + 31.41592653 \cdot i, \infty - 31.41592653 \cdot i]$$

If you add 'Real' as last parameter, then we obtain -9.4241... as the only one solution ...

Now let's try what we can do:

$$\#7: \text{mysolutions}\left(5 \cdot e^{-0.2 \cdot x} \cdot \sin(5 \cdot x) = \frac{1}{x}, x, -10, 10\right)$$

$$\#8: [-9.424133421, -8.79724213, -8.167184593, -7.540996268, -6.910050444, -6.284996034, -5.65258199, \\ -5.02945696, -4.39444978, -3.774892193, -3.134774689, -2.522850526, -1.870228676, -1.280867648, \\ -0.5638286745, -0.2167283973, 0.2296115766, 0.5438226014, 1.296884646, 1.85356236, 2.539524499, \\ 3.11760029, 3.792477949, 4.37625373, 5.048338594, 5.632915089, 6.305621164, 6.888424393, \\ 7.563885608, 8.14303882, 8.823010163, 9.396807952]$$

31 sec

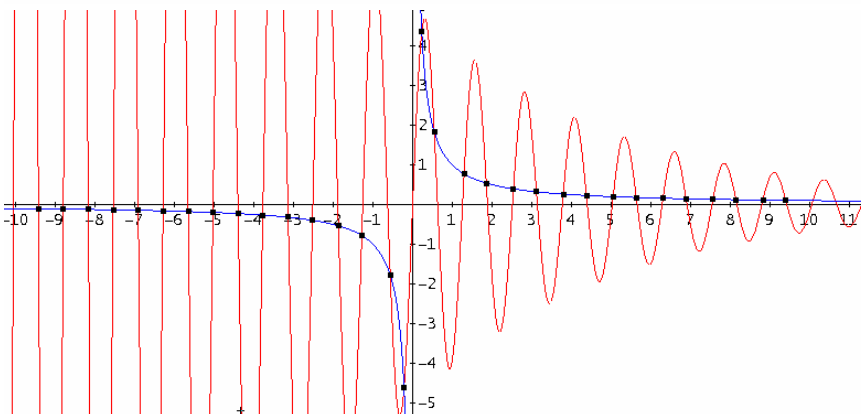
$$\#11: \text{N_SOLUTIONS}\left(5 \cdot e^{-0.2 \cdot x} \cdot \sin(5 \cdot x) = \frac{1}{x}, x, -10, 10\right)$$

$$\#12: [-9.424133421, -8.797242130, -8.167184593, -7.540996268, -6.910050444, -6.284996034, -5.652581990, \\ -5.029456960, -4.394449780, -3.774892193, -3.134774689, -2.522850526, -1.870228676, -1.280867648, \\ -0.5638286745, -0.2167283973, 0.2296115766, 0.5438226014, 1.296884646, 1.853562360, 2.539524499, \\ 3.117600290, 3.792477949, 4.376253730, 5.048338594, 5.632915089, 6.305621164, 6.888424393, \\ 7.563885608, 8.143038820, 8.823010163, 9.396807952]$$

7.4 sec

$$\#13: \text{sols} := \text{N_SOLUTIONS}\left(5 \cdot e^{-0.2 \cdot x} \cdot \sin(5 \cdot x) = \frac{1}{x}, x, -10, 10\right)$$

$$\#14: \text{VECTOR}\left(\left[x, \frac{1}{x}\right], x, \text{sols}\right)$$



I took some other equations and finally I must surrender and admit that Peter's tool is much better than mine, congratulations.

The nice thing is that Manuel Garcia's request on contour plots lead us to fine results. We hope that some of you will find them useful for their work.

Find intersection points of a parametric 3D ellipsoid and a 3D parametric straight line

Bill Wilburn, USA

```
solutions=N[Solve[x==4 Cos[s] Sin[t]&&y==3 Sin[s] Sin[t]&&z==2 Cos[t]
&&x==u+1&&y==3 u&&z==u+1,{x,y,z,s,t,u}]]
```

Solve::ifun: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. More...

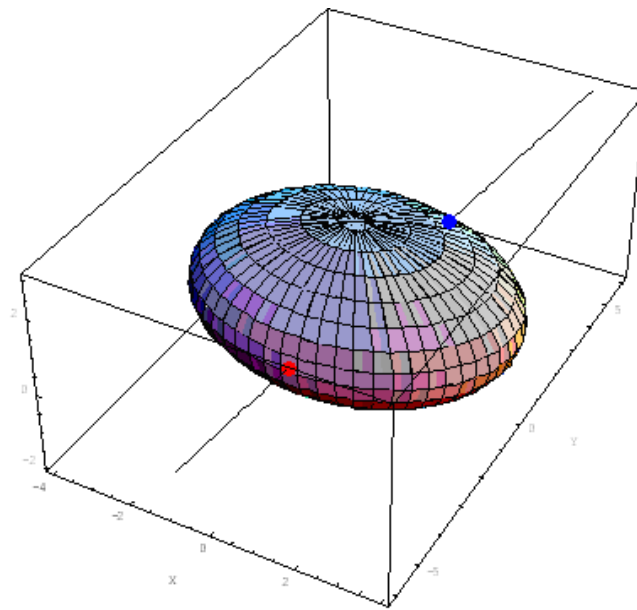
```
{{x→0.,y→-3.,z→0.,u→-1.,t→-1.5708,s→1.5708},{x→0.,y→-3.,z→0.,
u→-1.,t→1.5708,s→-1.5708},{x→1.52381,y→1.57143,z→1.52381,u→0.52381,
t→-0.704547,s→-2.19959},{x→1.52381,y→1.57143,z→1.52381,u→0.52381,
t→0.704547,s→0.942}}
```

```
{x,y,z}/. solutions
```

```
{{0.,-3.,0.},{0.,-3.,0.},
{1.5238095238095237,1.5714285714285714,1.5238095238095237},
{1.5238095238095237,1.5714285714285714,1.5238095238095237}}
```

```
plot1=ParametricPlot3D[{4Cos[s] Sin[t],3Sin[s] Sin[t],2Cos[t]},
{ s,-π,π},{ t,-π,π},Axes→True,AxesLabel→{"X","Y","Z"}];
plot2=ParametricPlot3D[{u+1,3 u,u+1},{u,-2,2}];
plot3=Show[Graphics3D[{RGBColor[1,0,0],PointSize[0.025],
Point[{0,-3,0}]}]];
plot4=Show[Graphics3D[{RGBColor[0,0,1],PointSize[0.025],
Point[{1.52380952380952,1.57142857142857,1.52380952380952}]}]];
Show[{plot1,plot2,plot3,plot4}]
```

Null



Parameter form:

#1: $[x_ := 4 \cdot \cos(s) \cdot \sin(t), y_ := 3 \cdot \sin(s) \cdot \sin(t), z_ := 2 \cdot \cos(t)]$

Implicit form:

#2:
$$\frac{x_^2}{16} + \frac{y_^2}{9} + \frac{z_^2}{4}$$

#3: 1

#4:
$$\text{SOLUTIONS} \left(\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} = 1 \wedge x = u + 1 \wedge y = 3 \cdot u \wedge z = u + 1, [x, y, z, u] \right)$$

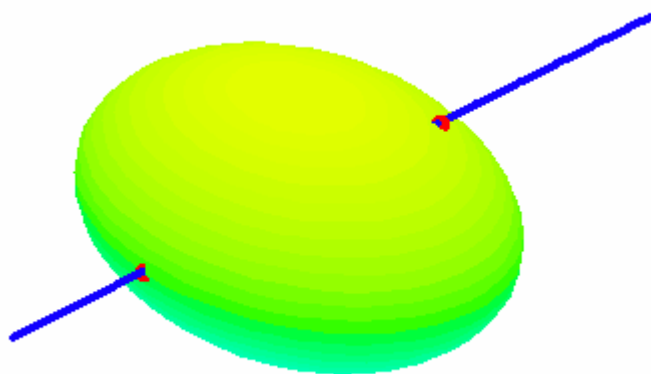
#5:
$$\begin{bmatrix} 0 & -3 & 0 & -1 \\ \frac{32}{21} & \frac{11}{7} & \frac{32}{21} & \frac{11}{21} \end{bmatrix}$$

#6:
$$\begin{bmatrix} [0, -3, 0] \\ \left[\frac{32}{21}, \frac{11}{7}, \frac{32}{21} \right] \end{bmatrix}$$

#7:
$$\begin{bmatrix} [0, -3, 0] \\ [1.523809523, 1.571428571, 1.523809523] \end{bmatrix}$$

#8: $\text{VECTOR}([u + 1, 3 \cdot u, u + 1], u, -3, 3, 0.01)$

I use the parameter form for the 3D-Plot (no mesh lines) and a similar trick as in the contribution on "Thick Lines" to plot the straight line as a thick line (Point Size Small is sufficient). The intersection points have Size Large.



We can do it in another way, too:

Substitute $\sin(t) = v$ and $\sin(s) = w$ and then square the components of line and surface.

The system is now consisting of polynomials only and can be solved by DERIVE.

#9: $\text{SUBST}([x_ := 4 \cdot \cos(s) \cdot \sin(t), y_ := 3 \cdot \sin(s) \cdot \sin(t), z_ := 2 \cdot \cos(t)], [s, t], [\text{ASIN}(v), \text{ASIN}(w)])$

#10:
$$[x_ := 4 \cdot w \cdot \sqrt{1 - v^2}, y_ := 3 \cdot v \cdot w, z_ := 2 \cdot \sqrt{1 - w^2}]$$

#11:
$$\text{SOLVE}([(u + 1)^2 = x_^2, (3 \cdot u)^2 = y_^2, (u + 1)^2 = z_^2], [u, v, w])$$

#12:
$$\left[u = -1 \wedge v = 1 \wedge w = 1, u = -1 \wedge v = 1 \wedge w = -1, u = -1 \wedge v = -1 \wedge w = 1, u = -1 \wedge v = -1 \wedge w = -1, u = \frac{11}{21} \wedge v = \frac{11 \cdot \sqrt{185}}{185} \wedge w = \frac{\sqrt{185}}{21}, u = \frac{11}{21} \wedge v = \frac{11 \cdot \sqrt{185}}{185} \wedge w = -\frac{\sqrt{185}}{21}, u = \frac{11}{21} \wedge v = -\frac{11 \cdot \sqrt{185}}{185} \wedge w = \frac{\sqrt{185}}{21}, u = \frac{11}{21} \wedge v = -\frac{11 \cdot \sqrt{185}}{185} \wedge w = -\frac{\sqrt{185}}{21} \right]$$

#13:
$$\text{VECTOR}\left([u + 1, 3 \cdot u, u + 1], u, \left[-1, \frac{11}{21}\right]\right)$$

#14:
$$\begin{bmatrix} 0 & -3 & 0 \\ 1.523809523 & 1.571428571 & 1.523809523 \end{bmatrix}$$

But there is also a “Black Box” – way, which doesn’t need more or less tricky manipulations. Students should know that there is a Newton Method for solving “uncomfortable equations” and they should learn that this method can be extended to more dimensions. In the DERIVE Online Help we can find via the index item ‘system of equations’ a reference to `Equation-Solving.mth` and there is a description of `NEWTONS(u,x,x0)` as a tool for approximative numerical solving of nonpolynomial systems. Let’s try:

#15: `NEWTONS([u + 1 - 4.COS(s).SIN(t), 3.u - 3.SIN(s).SIN(t), u + 1 - 2.COS(t)], [u, s, t], [1, 1, 1])`

#16:
$$\begin{bmatrix} 1 & 1 & 1 \\ 0.5643320325 & 0.9712704229 & 0.7125703502 \\ 0.5240209059 & 0.9423926175 & 0.7044230923 \\ 0.5238095638 & 0.9420000068 & 0.7045473966 \\ 0.5238095238 & 0.9420000403 & 0.7045474184 \\ 0.5238095238 & 0.9420000403 & 0.7045474184 \end{bmatrix}$$

We can observe that the method converges to a solution: $u = 0.5238\dots$, $s = 0.9420\dots$ and $t = 0.7045\dots$

To find another solution we have to change the initial values (which is now an initial vector for u , s , and t):

#17: `NEWTONS([u + 1 - 4.COS(s).SIN(t), 3.u - 3.SIN(s).SIN(t), u + 1 - 2.COS(t)], [u, s, t], [-1, -1, -1])`

leads to the same solution, but:

#19: `NEWTONS([u + 1 - 4.COS(s).SIN(t), 3.u - 3.SIN(s).SIN(t), u + 1 - 2.COS(t)], [u, s, t], [0, 0, 0])`

needs about 14 iterations to converge for the second solution.

#20:
$$\begin{bmatrix} -1 & -4.71238898 & 4.71238898 \\ -1 & -4.71238898 & 4.71238898 \\ -1 & -4.71238898 & 4.71238898 \end{bmatrix}$$

So we obtained the same intersection points as above.

#21: `VECTOR([u + 1, 3.u, u + 1], u, [0.5238095238, -1])`

#22:
$$\begin{bmatrix} 1.523809523 & 1.571428571 & 1.523809523 \\ 0 & -3 & 0 \end{bmatrix}$$

We could refine `NEWTONS` to make the output more friendly. In fact we are only interested in the last rows to see if and where the method is converging.

```
APPEND([x, y, z, u, s, t]], (REVERSE(NEWTONS([x - (u + 1), x - 4.COS(s).SIN(t), y - 3.u, y -
```

```
#26:
```

```
3.SIN(s).SIN(t), z - (u + 1), z - 2.COS(t)], [x, y, z, u, s, t], [1, 1, 1, 1, 1, 1]))
```

```
[1, 2, 3]
```

```
#27:
      x          y          z          u          s          t
1.523809523  1.571428571  1.523809523  0.5238095238  0.9420000403  0.7045474184
1.523809523  1.571428571  1.523809523  0.5238095238  0.9420000403  0.7045474184
1.523809563  1.571428691  1.523809563  0.5238095638  0.9420000068  0.7045473966
```

We don't only see the convergence, but also all solutions – 6 equations and 6 unknowns. For the other solution you have to take [0,0,0,0,0,0] as initial vector.

I sent this treatment to Wilburn and a very few days after I received another mail with an improved (= more complicated) version of his problem. The centre of the surface moved out of the origin:

```
Clear[sol]
```

```
sol=NSolve[x==1.414213562 Cos[s] Sin[t]-1.719669914 Sin[s]
Sin[t]+1.478397839 Cos[t]&&y==2.449489742 Cos[s] Sin[t]+2.217596759 Sin[s]
Sin[t]+0.5606601717 Cos[t]&&z==2.828427124 Cos[s] Sin[t]+1.060660171 Sin[s]
Sin[t]+1.224744871 Cos[t]&&x==u+1&&y==3 u&&z==u+1,{x,y,z,s,t,u}]
```

Solve::ifun: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. More...

```
{x→-0.02864356966104064,y→-3.085930708983122,z→-0.02864356966104064,
u→-1.0286435696610405,t→-2.039686505060742,s→1.0160172805323466},
{x→-0.02864356966104064,y→-3.085930708983122,z→-0.02864356966104064,
u→-1.0286435696610405,t→2.039686505060742,s→-2.1255753730574467},
{x→1.2890643961420396,y→0.8671931884261188,z→1.2890643961420396,
u→0.2890643961420396,t→-0.12105797768231857,s→-1.7273026137850234},
{x→1.2890643961420396,y→0.8671931884261188,z→1.2890643961420396,
u→0.2890643961420396,t→0.12105797768231857,s→1.414290039804777}
```

```
{x,y,z},{s,t},{u} /. {sol[[1]],sol[[3]]}
```

```
{-0.0286436, -3.08593, -0.0286436} {1.01602, -2.03969} {-1.02864}
{1.28906, 0.867193, 1.28906} {-1.7273, -0.121058} {0.289064}
```

```
aa=ParametricPlot3D[{1.414213562 Cos[s] Sin[t]-1.719669914 Sin[s]
Sin[t]+1.478397839 Cos[t], 2.449489742 Cos[s] Sin[t]+2.217596759 Sin[s]
Sin[t]+0.5606601717 Cos[t], -2.828427124 Cos[s] Sin[t]+1.060660171 Sin[s]
Sin[t]+1.224744871 Cos[t],
FaceForm[Hue[s/Pi]]},{s,-Pi,Pi},{t,-Pi,Pi},Lighting → False,
Axes → True, AxesLabel → (StyleForm[#, "Subtitle", FontSize → 12,
FontWeight → "SemiBold"] & /@ {"X", "Y", "Z"}),
PlotLabel → StyleForm["Geometry", FontFamily → "Times",
FontSize → 18, FontWeight → "Bold"]];
bb=ParametricPlot3D[{u+1, 3u, u+1}, {Blue, AbsoluteThickness[2]}], {u, -2, 2}];
cc= Show@Graphics3D[{{AbsolutePointSize[10], Red, Point[{x,y,z] /. sol[[1]]}},
{AbsolutePointSize[10], Red, Point[{x,y,z] /. sol[[3]]}}];
Show[aa, bb, cc]
```

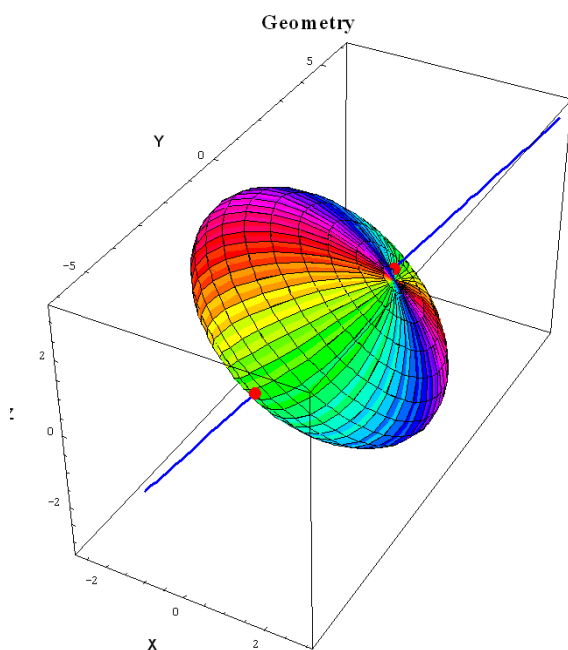
We have studied our lection in the first example, so we can start immediately solving the problem – now using immediately the Black Box (Try to do without!):

```
#28: e112 := [1.414213562.COS(s).SIN(t) - 1.719669914.SIN(s).SIN(t) + 1.478397839.COS(t),
            2.449489742.COS(s).SIN(t) + 2.217596759.SIN(s).SIN(t) + 0.5606601717.COS(t), -
            2.828427124.COS(s).SIN(t) + 1.060660171.SIN(s).SIN(t) + 1.224744871.COS(t)]

#29: line2 := [u + 1, 3.u, u + 1]
sys2 := VECTOR(e112 - line2, k, 3)
#30:
#31: sys2 := [1.414213562.COS(s).SIN(t) - 1.719669914.SIN(s).SIN(t) + 1.478397838.COS(t) - u - 1,
            2.449489741.COS(s).SIN(t) + 2.217596759.SIN(s).SIN(t) + 0.5606601717.COS(t) - 3.u, -
            2.828427124.COS(s).SIN(t) + 1.060660171.SIN(s).SIN(t) + 1.22474487.COS(t) - u - 1]

#32: (REVERSE(NEWTONS(sys2, [u, s, t], [0, 0, 0]))) [1, 2]
#33: [ -1.028643569  -2.125575373  2.039686505 ]
      [ -1.028643569  -2.125575373  2.039686505 ]
#34: (REVERSE(NEWTONS(sys2, [u, s, t], [1, 1, 1]))) [1, 2]
#35: [ 0.2890643961  1.414290039  0.1210579776 ]
      [ 0.2890643961  1.414290039  0.1210579776 ]
#36: u_s := [-1.028643569, 0.2890643961]
#37: VECTOR([line2], u, u_s)
#38: [ [-0.028643569, -3.085930707, -0.028643569] ]
      [ 1.289064396, 0.8671931882, 1.289064396 ] ]
```

Mathematica Graph



DERIVE Graph (Greyscale)

