Calculus Concepts Using Derive For Windows

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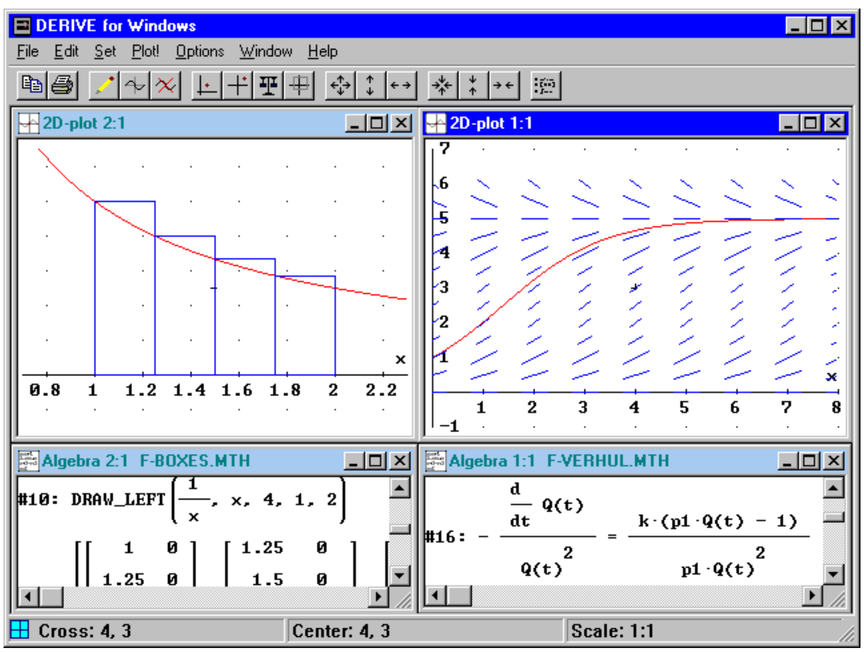

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`http://www.math.hawaii.edu/CalcLabBook/`

and to see our class web page containing our syllabus, assignments and general information for students see

`http://www.math.hawaii.edu/lab/`

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Chapter 0

Introduction and Derive Basics

0.1 Overview

In this course you will learn to use the computer mathematics program DERIVE. This program, along with others such as Maple and Mathematica, are very powerful tools for doing calculus. They are capable of doing exact computations with arbitrary precision. This means that you can work with numbers of any size or number of decimal places (most spreadsheets only use 10-20 significant digits). These programs can simplify mathematical expressions by canceling common factors and doing other algebraic operations. They can do symbolic calculus such as differentiation and integration, solve equations and factor polynomials. When possible these programs solve these problems exactly and when exact solutions do not exist, such as factoring high degree polynomials or integration of some non-polynomial expressions, then numerical methods are applied to obtain approximate results.

Probably the most important numerical technique is to graph and compare functions. This will be a key feature of the labs. Typically we will explore a topic by first graphing the functions involved and then trying to do symbolic calculus on them using the insight gained from the picture. If the problem is too difficult algebraically we then try numerical techniques to gain further insight into the problem. It is this combination of graphics, algebra and numerical approximation that we want to emphasize in these labs.

Calculus is a hard subject to learn because it involves many ideas such as slopes of curves, areas under graphs, finding maximums and minimums, ana-

lyzing dynamic behavior and so on. On the other hand, many computations involve algebraic manipulations, simplifying powers, dealing with basic trig expressions, solving equations and other techniques. Our goal is to help you understand calculus better by concentrating on the ideas and applications in the labs and let the computer do the algebra, simplifying and graphing.

Another important goal of the lab is to teach you a tool which can used from now on to help you understand advanced work, both in mathematics and in subjects which use mathematics. There are many features such as matrices and vector calculus which we will not discuss but can be learned later as you continue with your studies in mathematics, physics, engineering, economics or whatever. Any time you have a problem to analyze you can use the computer to more thoroughly explore the fundamental concepts of the problem, by looking at graphs and freeing you from tedious calculations.

This chapter contains a brief introduction on how to use Derive. We suggest you sit down at the computer and experiment as you look over the material. Derive is very easy to learn thanks to its system of menus. The few special things you need to remember are discussed below and can also be found using the help feature in Derive.

0.2 Starting Derive

The computers in our labs are IBM-PCs running the Window 95 operating system. We will describe how to use the DfW (Derive for Windows) software in the Window 95 environment. To start DfW look for

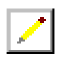
on the desktop and double click it.

In DfW we use the drop down menus on the top strip or else click an appropriate button. If you move the cursor onto a button and leave it there a brief explanation of what the button does will appear. All possible options can be found on the drop down menus but the buttons provide a quick way of doing most common operations. For example, to enter a mathematical expression you click the  button which represents a pencil. Alternately, you click the Author menu and then click Expression. In this manual we will indicate that two step combination by simply Author/Expression.

In this manual we use a typewriter like font, eg., `a(b + c)` to indicate something you might type in. We use a sans serif font for special keys on the

keyboard like Enter (the return key) and Tab. Most of DERIVE has easy to use menus and buttons which we will described below. It only takes a few minutes of practice to become capable at using DfWẎou can skim over this section now or then come back to it later as the need arises.

0.3 Entering an Expression

After clicking the  button, you enter a mathematical expression, i.e., you type it in and then press the Enter key or else click OK. You enter an expression using the customary syntax: addition `+`-key, subtraction `−`-key, division `/`-key, powers `^`-key and multiplication `*`-key (however; multiplication does *not* require a `*`, i.e., `2x` is the same as `2*x`). DERIVE then displays it on the screen in two-dimensional form with raised superscripts, displayed fractions, and so forth. You should always check to make sure the two-dimensional form agrees with what you thought you entered (see **Editing** below to see how to correct typing errors). Table 1 gives some examples.

If you get a syntax error when you press enter (or click OK) the problem is usually mismatched parentheses. Carefully check that each left parenthesis is matched with a corresponding right one. Also be careful to use the round parentheses and not the square brackets since they are used for vector notation; see Section 0.14 on page 18.

Note from (3) and (4) and from (6) and (7) of Table 1 that it is sometimes necessary to use parentheses. Also note in (8), that to get the fraction you want, it is necessary to put parentheses around the numerator and denominator. See what happens if you enter (8) without the parentheses. Also try entering some expressions of your own. There are two ways to enter square roots. One way is using the 0.5 or 1/2 power as in (9) and the other is to enter the special square root character as in (10).

0.4 Special Constants and Functions

In DfW all the special characters are on the author form and you just click on them to enter them in an expression. There are also key equivalent such as `pi` for π and `#e` for Euler's constant[1] e which is displayed by DERIVE as $\hat{e}$.

[1]Leonhard Euler (*oi'lar*) was a 18^{th} century Swiss mathematician.

Table 1:

	You enter:	You get:
(1)	`25`	25
(2)	`x^2`	x^2
(3)	`a^2x`	a^2x
(4)	`a^(2x)`	a^{2x}
(5)	`sin x`	$\sin(x)$
(6)	`sin a x`	$\sin(a)x$
(7)	`sin(a x)`	$\sin(ax)$
(8)	`(5x^2 - x)/(4x^3 - 7)`	$\frac{5x^2-x}{4x^3-7}$
(9)	`(a + b)^(1/2)`	$(a+b)^{1/2}$
(10)	`√ (a + b)`	$\sqrt{}(a+b)$

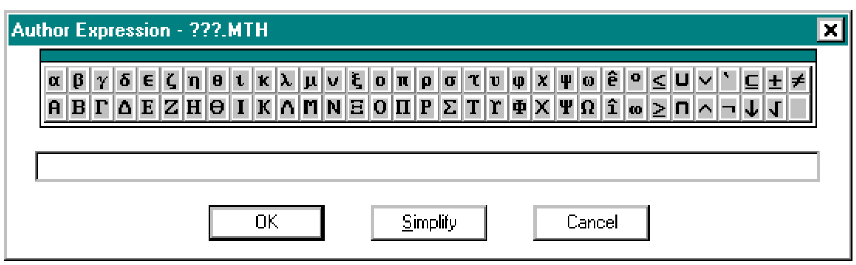
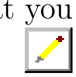


Figure 0.1: Author entry form with special symbols

It is important to distinguish $\hat{e}$ from just e. DERIVE takes e to just be some constant like a.

To get the functions $\tan^{-1} x = \arctan(x)$, $\sin^{-1} x = \arcsin(x)$, etc., you type `atan x` and `asin x`. If you forget how a particular function is entered just use the Help menu.

0.5 Editing

Suppose that you author an expression, click OK and then observe that you typed something wrong. To enter a correction you would click the  button again and then *click the right mouse button*. A menu opens up with several options, one of which is Insert Expression. Clicking this option puts the highlighted expression of the current algebra window into the author box. You edit this expression as you would in any windows program. That is you position the cursor either by clicking or using the arrow keys. By highlighting or selecting a subexpression and typing you replace the selected text with the new text. One can use the Edit/Copy Expressions menu or Cntl-C to place a highlighed expression from any algebra window onto the clipboard and then in the authoring form right click the mouse and click Paste to copy the clipboard contents. The simpler method of just right clicking the mouse and then Insert is the best way as long as your expressions are in a single window. There is also an option for inserting the expression enclosed in parenthesis. A key equivalent to these techniques is to press either the F3 or the F4 key.

You select or highlight expressions in the algebra window by clicking on them. For more complicated expressions you can click several times until the desired subexpression is selected. This requires a little practice but you can, for example, select the $x + 2$ part of the expression $\sin \frac{x^2}{x(x+2)}$ by clicking on it 4 times (each click takes you deeper into the expression).

The displayed expressions are numbered. You can refer to them as `#n`. So, for example, with the expressions in Table 1, you could get $\sin(x)/x^2$ by Authoring `#5/#2`.

When you start DERIVE it is in a character mode. This means it treats each single character as a variable, so if you type `ax` DERIVE takes this to be a times x. This mode is what is best for calculus. The exception to this are the functions DERIVE knows about. If you type `xsinx`, DERIVE knows you want $x \sin(x)$. Actually on the screen you will see $x\,\mathrm{SIN}(x)$: DERIVE displays all variables in lower case and all functions in upper case.

0.6 Simplifying and Approximating

After you enter an expression, DERIVE displays it in two-dimensional form, but does not simplify it. Thus, integrals are displayed with the integral sign and derivatives are displayed using the usual notation. To simplify (that is

Table 2: Special Keys and Function Names

Expression/Action	Type:	Menu:
e	`#e`	Author entry form
π	`pi`	Author entry form
∞, $-\infty$	`inf`, `-inf`	Author entry form
The square sign: $\sqrt{x}$	`sqrt(x)`	Author entry form
$\ln x$, $\log_b x$	`ln x` , `log(b, x)`	
Inverse trigonometric functions	`asin x`, `atan x`, *etc.*	
$\dfrac{d}{dx}f(x)$	`dif(f(x), x)`	Calc/Differentiate
$\dfrac{d^n}{dx^n}f(x)$	`dif(f(x), x, n)`	Calc/Differentiate
$\int f(x)\,dx$	`int(f(x), x)`	Calc/Integrate
$\int_a^b f(x)\,dx$	`int(f(x), x, a, b)`	Calc/Integrate
Simplify an expression		Simplify
Approximate		Simplify/Approx
Cancel a menu choice	Esc-*key*	
Move around in a menu	Tab-*key*	
Change highlighted expression	▲, ▼-*key*	Click expression
Insert highlighted expression	F3 , F4 with ( )'s	Right mouse button

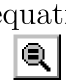
evaluate) the expression, click the = button. The alternate method is the Simplify/Basic drop down menu.

Derive uses exact calculations. If you Author the square root of eight, $\sqrt{8}$ will be displayed in the algebra window. If you simplify this, you get $2\sqrt{2}$. If you want to see a decimal approximation, you click the ≈ button. See Figure 0.5 on page 15 for several examples. The number of decimal places used can be changed to any number. You choose Declare/Algebra State/Simplification and then reset the number of decimals places. This results in a change in the State variables for Derive and you will be prompted on whether you want to save these changes when you exit the program. Since you can't change files on the system directory you should click No.

An alternate way to do this for a single computation is to select the Simplify menu. Then, choose Approximate from the drop down menus and enter a new number of decimals. The only trouble with this method is that if you save your file the extra decimals will be ignored unless you set the Output decimal places appropriately. When you open the file later you will also need to reset the Output decimal place accuracy.

0.7 Solving Equations

An important problem is to find all solutions to the equation $f(x) = 0$. If $f(x)$ is a quadratic polynomial such as $x^2 - x - 2$, then this can be done using the quadratic formula or by factoring. To factor you choose Simplify/Factor from the menu bar and click Simplify on the entry form. The result is $(x + 1)(x - 2)$. This means that the roots of $f(x)$ are $x = -1$, 2, i.e., these are the only solutions to $f(x) = 0$.

We can also do this by using the `SOLVE` function. To do this we highlight the equation, say $x^2 - x - 2$ (it's assume to be equal to zero), and click the  . If you forget the function of a button just hold the cursor on it and a brief explanation will appear. An alternate method is to choose Solve/Algebraically from the drop down menu. The quadratic formula is used to solve for the roots so it is possible the answer will involve square roots (and even complex solution, e.g., $x^2 + 1$ has no *real* roots but it does have two complex ones, namely, $x = \pm i$).

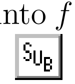
If $f(x)$ is not a quadratic polynomial then Derive may not be able to factor it; nevertheless, it may be able to solve the equation $f(x) = 0$. As

an example, $\sin x = 0$ has infinitely many solution $x = m\pi$ where m is any integer. If we use DERIVE to solve this equation it gives the 3 solutions corresponding to $m = -1, 0, 1$ (these are the principle solutions and all others are obtained by adding or subtracting multiples of 2π).

Finally, the simple equation $\sin x - x^2 = 0$ cannot be solved exactly in DERIVE although it is obvious that $x = 0$ is one solution and by viewing the graph we see another one with $x \approx 1$. In order to approximate this solution we need to choose Solve/Numerically. We will then be asked for a range of x's (initially it is the interval $[-10, 10]$). Since we have (at least) 2 solutions in $[-10, 10]$, we should restrict the interval to say $[.5, 1]$ which seems reasonable based on the graphical evidence. The result is that DERIVE gives the solution $x = .876626$. We will discuss how this computation is done later in Chapter 5.

Note that Solve/Numerically will only give one solution (or none if there are none) even if the interval you choose cantains several solutions. To find additional solutions you need to use Solve/Numerically again but with an interval avoiding the first solution.

0.8 Substituting

If you have an expression like $\sqrt{x^2+1}/x$ and you want to evaluate this with say $x = 3$ or if you solved an equation $f(x) = 0$ and you want to substitute that value of x back into $f(x)$, you start by highlighting the desired expression. Next you click the 

button and the substitution form opens up. You need to fill in the substitution value so you would just type 3 in the first example. On the other hand, if the substitution value is large, say lots of digits or some other complicated expression in the algebra window, the easiest way is to move the form out of the way (just hold down the left mouse button in the top strip and drag to another location) and select the desired expression by clicking on it. Then, paste it into the form by right clicking and choosing Insert. If there happen to be other variables in the expression you may have to change the variable in the variable list box.

0.9 Calculus

This menu item is very important for us. After choosing the Calculus drop down menu, you get a submenu with Limit, Differentiate, Taylor series, Integrate, Sum, Product, and Vector. There are also buttons for most of these operators. After you have authored an expression, you can differentiate it by either clicking the ∂ button or choosing Calculus/Differentiate from the drop down menu. The form will have entries for what variable to use and how many times to differentiate, but it usually guesses right so you can just click OK. Then simplify.

To integrate an expression, first author it or highlight it if it is already in the algebra window, then either click the ∫ button or else choose Calculus/Integrate. The form will have entries for what expression to integrate; it will guess you want to integrate the highlighted expression. It will have an entry for what variable you what to integrate with; again it will probably guess right. It will also have entries for the limits of integration. If you want an indefinite integral, just click the appropriate button and click OK. For a definite integral click the appropriate button, type in the upper/lower limit, then click OK. See Figure 0.2 on the following page for several examples using Differentiate and Integrate on the Calculus menu.

The options Calculus/Limits is similar to the above. To find

$$\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$$

you enter the expression, then either click lim or choose Limits from the Calculus menu. You fill in the variable (which is x) and the limit point which is -1 since $x \to -1$. Then click = or choose Simplify to get the answer. In a similar manner DERIVE does summation and product problems. Special notations are used; namely,

$$\sum_{i=1}^{n} a_i = a_1 + \cdots + a_n \quad \text{and} \quad \prod_{i=1}^{n} a_i = a_1 \cdots a_n.$$

Let us discuss the summation notation which may be new to you. If $a_1, \ldots, a_n$ are numbers then

$$\sum_{i=1}^{n} a_i = a_1 + \cdots + a_n.$$

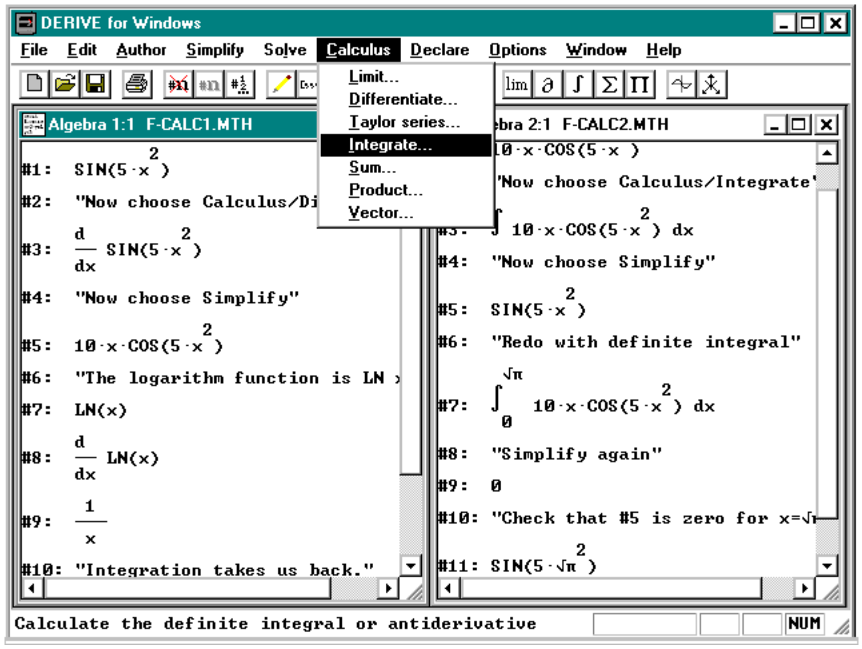


Figure 0.2: Using the Calculus menu

The symbol on the left, $\sum_{i=1}^{n} a_i$, is read as "the sum of a_i as i runs from 1 to n." Often a_i is a formula involving i. So

$$\sum_{i=0}^{5} i^{\,2} = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55.$$

You can do this computation in DERIVE by clicking the $\boxed{\Sigma}$ or using the Sum option on the Calculus menu. Just author `i^2` then click $\boxed{\Sigma}$. Fill in the required variable i along with the starting value 0 and end value 5. Simplify to get 55. As an interesting aside, edit the above sum and have DERIVE Simplify $\sum_{i=1}^{n} i$ to get the formula:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

This formula is used in many calculus texts to evaluate certain Riemann sums.

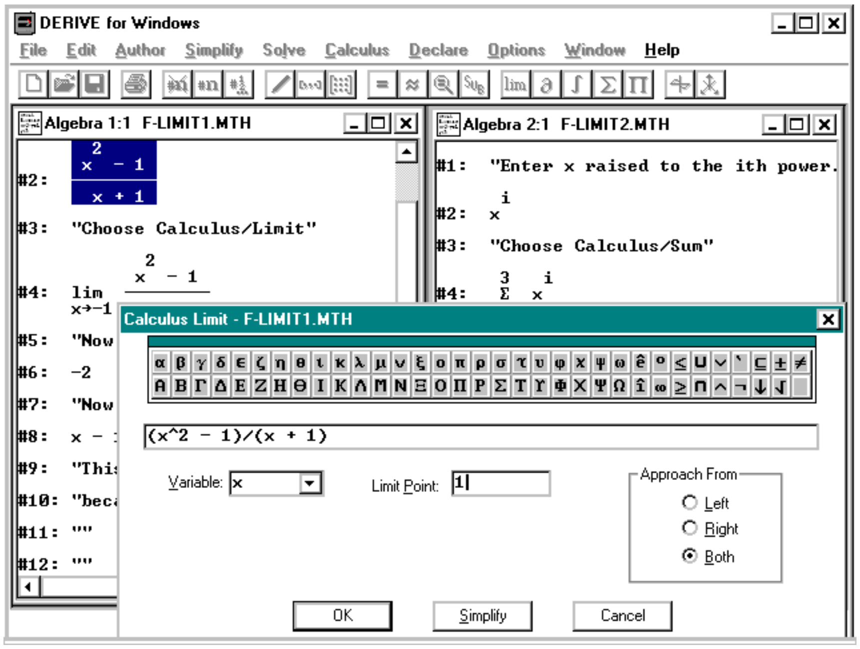


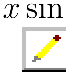
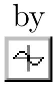
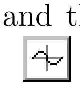
Figure 0.3: Examples of Limits, Products and Sums

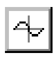
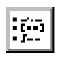
See Figure 0.3 for some examples. Note that in Figure 0.3

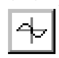
$$\prod_{i=1}^{3} x^i = x \cdot x^2 \cdot x^3 = x^6.$$

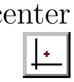
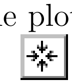
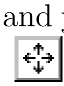
The option Calculus/Taylor will be explained in Chapter 7.

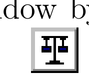
0.10 Plotting

Supposed you want to graph the function $x \sin x$. You simply author the expression, by clicking the pencil button  , to be plotted and then click the  button. A plot window will then opens up and the icon-bar will change to a new set of buttons. You then click the  button again (it's position is different in the plot window) and the graph will be drawn. There are several different ways to view the algebra and plot window together. The one we used to produce the pictures in the manual is to first

select the algebra window (if you are currently in the plot window you can go to the algebra window by clicking the  button) and then choose Window/Tile Vertically from the menus. This will split the screen into two windows: an algebra window on the left and a plotting window on the right. These windows each have a number in their upper left hand corner. You can have several plot windows associated with a single algebra window but you cannot plot together expressions from different algebra windows. You can switch windows by either clicking the top strip of the window or clicking the  or  buttons. Actually you can click anywhere in the window to select it but the top strip avoids changing the highlighted expression in the algebra window or moving the cross in the plot window.

You can plot several functions in the same plot window. Move to the algebra window, highlight the expression you want to plot, switch to the plot window and then click the  in the plot window. Now both expressions will be graphed. You can plot as many as you want this way. The plot window also has a menu option, Edit/Delete Plot, for removing some or all of the expressions to be plotted. Pressing the Delete-key also removes the current plot.

When you plot, there is a small crosshair in the plot window, initially at the $(1, 1)$ position. You can move the cross around using the arrow keys or by clicking at a new location. The coordinates of the cross are give at the bottom of the screen. This is useful for such things as finding the coordinates of a maximum or a minimum, or where two graphs meet. In order to center the graph so that the cross is in the center of the window, click the  button. This is useful for zooming in and out to get a better view of the graph. There are several buttons for doing this in the plot window. Take a look and you will see a button for zooming in, namely , and for zooming out  and various ways of changing just the x-scale or just the y-scale. You should try clicking these buttons to see exactly what happens.

In general, these buttons change the scale of the plot window by either doubling or halving it. You can customize these by using the  button (that's a picture of a balance scale). Just click this button and fill out the form the appears with your own numbers. You can see the current scale at the bottom of the screen.

We mentioned above how to plot any number of graphs simultaneously by repeatedly switching between the algebra window and the graphics window.

Another technique for plotting three or more functions is to plot a vector of functions. This just means authoring a collection of functions, separated by commas and surrounded by square brackets. For example, plotting the expression `[x, x^2, x^3]` will graph the three functions: x, x^2, and x^3. In order to plot a collection of individual points one enters the points as a matrix, for example authoring the expression `[[-2,-2], [0,-3], [1,-1]]` and then plotting it will graph the 3 points: $(-2,-2)$, $(0,-3)$ and $(1,-1)$. A quick way of authoring a vector is to use the button and a quick way to enter a matrix is to use the button. One then just fills out the form that open up. So for example with the 3 points above we would click the matrix button and select 3 rows and 2 columns. The form will open up and we then fill in the 6 numbers above in the obvious order. You move between fields by either clicking or using the Tab-key.

When plotting points you have a choice of connecting the points with a line segment so that it appear like the graph of a function. You do this by choosing Options from the menu bar. There are lots of interesting items on this menu that will allow you to customize plotting colors, the size points are plotted, axes and so on. To connect points we choose Points and then check the Yes button. We can also modify the size of the points by clicking the appropriate button. See the Figure 0.4 on the following page where each of these techniques is demonstrated. The color of a plot is controlled by choosing Option/Plot Color and then making sections on the menu.

0.11 Defining Functions and Constants

If you Author `f(x)`, Derive will put $f\,x$ on the screen because it thinks both x and f are variables. If you wish to *define* $f(x) = x^2 + 2x + 1$, for example, you could Author `f(x) := x^2 + 2x + 1`. Note that we use := for assignments and = for equations. Alternately, you could choose Declare/Function, and then fill in the form with `f(x)` for the function name and `x^2 + 2x + 1` for its value. Derive will then enter this as above with the :=-sign. See Figure 0.5 on page 15.

Constant are treated just like functions except there are no arguments. In order to set $a = 2\pi$ for example you type `a := 2 pi`. Then, whenever you simplify an expression containing a, each occurrence is replaced with 2π.

In many problems you find it useful to have constant names with more

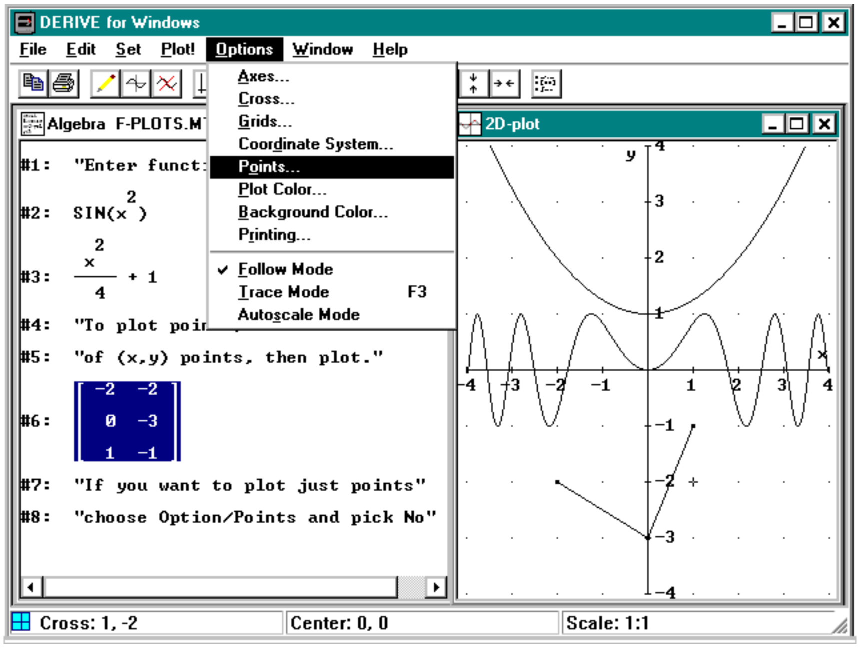


Figure 0.4: Using Plot for graphics

than one letter or symbol, which is the default in DERIVE. For example variables with names like `x1`, `y2`, etc. will be used frequently as are names like "gravity". This can be done by *declaring the variable*, for example, to use the variable $x1$ we author `x1:=`. Now any use of these letters will be treated as the single variable $x1$.

An interesting function defining technique is provided by the factorials. For $n = 1, 2, \ldots$ we define n-factorial, denoted by $n!$, as

$$n! = n \cdot (n-1) \cdots 2 \cdot 1 \qquad n = 1, 2, \ldots$$

and for completeness we define $0! = 1$. These numbers are important in many formulas, e.g., the binomial theorem. One observes the important *recursive* relationship $n! = n(n-1)!$ which gives the value of $n!$ in terms of the previous one $(n-1)!$. Thus, since $5! = 120$ we see immediately that $6! = 720$ without multiplying all 6 numbers together.

In DERIVE we can recursively define a function `F(n)` satisfying $F(n) = n!$ by simply typing

```
F(n) := IF(n=0, 1, n F(n-1))
```

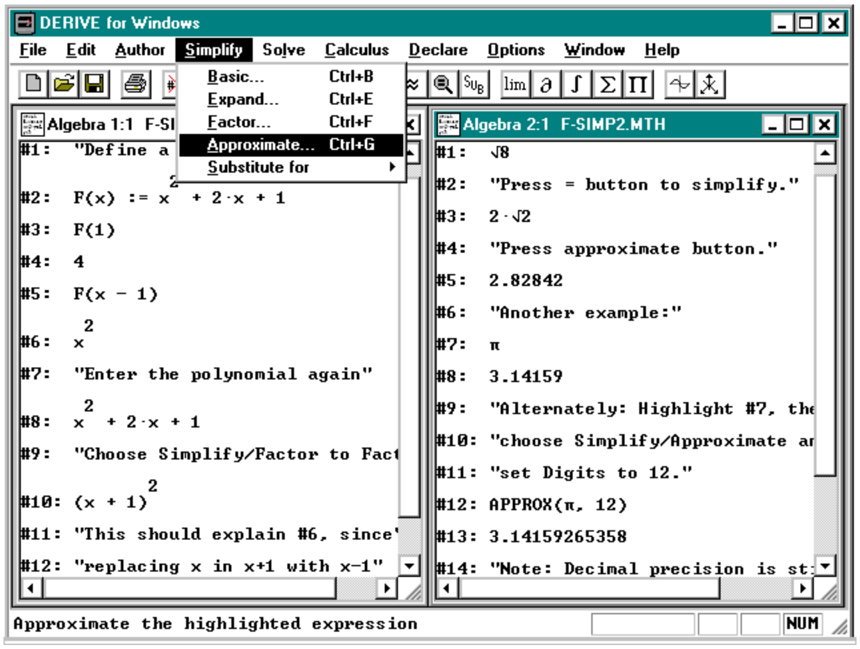


Figure 0.5: Examples of Declare, Simplify and approximating

where the properties of the DERIVE function `IF(test, true, false)` should be pretty obvious. The definition forces the function to circle back over and over again until we get to the beginning value at $n = 0$, i.e.,

$$F(n) = n \cdot F(n-1) = \cdots = n \cdot (n-1) \cdots 2 \cdot 1 \cdot F(0) = n!$$

We will give several other examples of this technique in the text.

0.12 Defining The Derivative Function

A common application of defining functions is to have $f(x)$ defined but the calculus problem requires a formula involving both $f(x)$ and $f'(x)$. For example, the equation of the tangent line at the point $(a, f(a))$ is given by $y = f(a) + f'(a)(x - a)$.

If you try to define the derivative of $f(x)$ by `g(x) := dif(f(x), x)` and then evaluate $g(2)$, you get `DIF(F(2), 2)`, which is not what you want. Of course we could also just compute the derivative and define `g(x) :=` to

be that expression. The advantage of defining it as a function is that if we change the definition of $f(x)$, then $g(x)$ will also change to the derivative of the new $f(x)$. Thus, we get to use the formula for more than one application.

Here's the correct way to define the derivative as a function: Start by Authoring `f(x):=` and we can enter the specific definition of $f(x)$ now or wait until later. Next, click the derivative ∂ button and enter `f(u)` in the form (note the variable is u not x). Select the Variable `u` and press OK. Now click the limit lim button (with the previous expression highlighted) and enter the Variable `u` and the Point `x`. Finally, Author `g(x):=` and insert the previous expression by right-clicking and selecting Insert.

The result is the expression `G(x):= LIM( DIF( F(u), u), u, x)`. Actually, you could have just Authored this expression directly but the syntax and the number of parentheses is a little confusing in the beginning so the above method is easier and probably faster. See all this worked out on page 40 in Chapter 2 where a more technical discussion of this issue is given.

0.13 Functions Described By Tables

In calculus functions are typically described by giving a formula like $f(x) = 2x^3 + 5$ but another technique is to describe the values restricted to certain intervals or with different formulas on different ranges of x-values. As an example, consider the function

$$f(x) = \begin{cases} 2x+1 & \text{for } x < 1 \\ x^2 & \text{for } 1 \leq x \leq 2 \\ 4 & \text{for } 4 < x \end{cases}$$

which defines a unique value $f(x)$ for each value of x. The problem is how do we define such a function using DERIVE?

One basic technique is to use the logical `IF` statement. The syntax is `IF(test, true, false)`. For example, if we enter and simplify `IF(1 < 2, 0, 1)` we get 0 whereas `IF(1 = 2, 0, 1)` simplifies to 1. Now our function above is entered as:

```
f(x):= IF(x < 1, 2x + 1, IF(x <= 2, x^2, 4))
```

Notice how we use nested `IF` statements to deal with the three conditions and that with four conditions even more nesting would be required. Now once

$f(x)$ has been defined we can make computations such as **S**implifying `F(1)` (should get 1), computing limits such as the right-hand limit $\lim_{x\to 1+} f(x)$ (should get x^2 evaluated at $x = 1$) or definite integrals using appro**X** to simplify. We can also plot $f(x)$ in the usual manner described in the previous section.

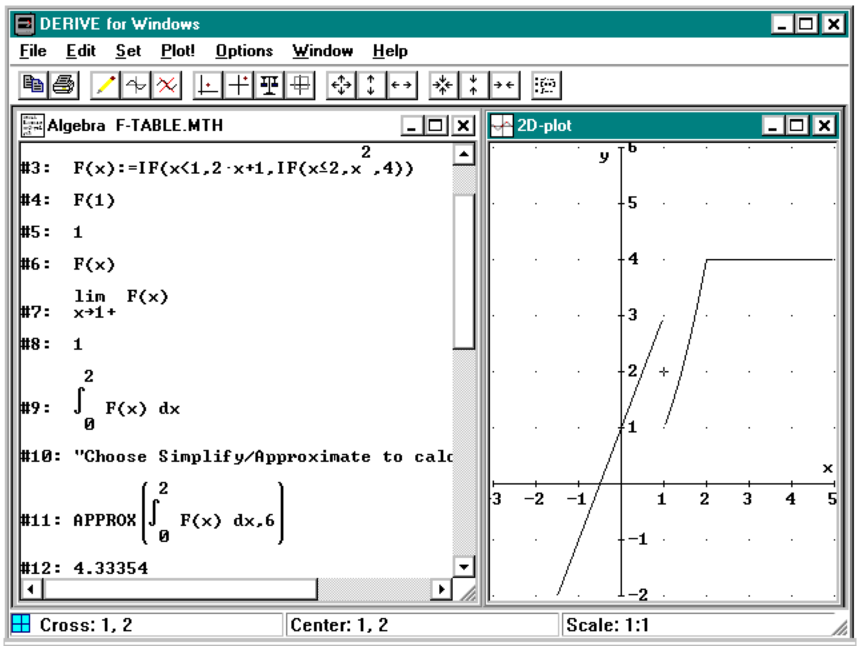


Figure 0.6: Functions defined by tables of expressions

Notice from Figure 0.6 that the function $y = f(x)$ is continuous at all $x \neq 1$. At $x = 1$, both left and right limits exist but they are not equal so the graph has a jump discontinuity.

As the number of table entries increases we are forced into using nested `IF` statements and the formulas become quite difficult to read and understand. An alternate approach is to use the DERIVE function `CHI(a,x,b)` which is simply

$$\text{CHI}(a,x,b) = \begin{cases} 0 & \text{for } x \leq a \\ 1 & \text{for } a < x < b \\ 0 & \text{for } b \leq x \end{cases}$$

Then except for $x = 1$ our function $f(x)$ above satisfies:

```
F(x):=(2x+1) CHI(-inf,x,1) + x^2 CHI(1,x,2) + 4 CHI(2,x,inf)
```

This technique works for graphing and limit problems and moreover gives the exact result at each point where the function is continuous.

0.14 Vectors

Vectors are quite useful in Derive, even for calculus. They are also useful in plotting. To enter the 3 element vector with entries a, b, and c, we can Author `[a, b, c]` directly. It is important to note the square brackets which are used in Derive for vectors; commas are used to separate the elements. An easier approach is just click the  button and fill in the three values on the vector input form.

Derive also provides a useful function for constructing vectors whose elements follow a specific pattern. The `vector` function is a good way to make lists and tables in Derive. For example, if you Author `vector(n^2, n, 1, 3)`, it will simplify to `[1, 4, 9]`. The form of the `vector` function is `vector(u,i,k,m)` where `u` is an expression containing `i`. This will produce the vector $[u(k), u(k+1), \ldots, u(m)]$. You can also use the Calculus/Vector menu option to create a vector. So, for example, to obtain the same vector as before, you start by authoring `n^2`. Now choose Calculus/Vector and fill in the form setting the Variable to n (not x), the start value to 1, the end value to 3 and the step size to 1 (that's the default value).

A table (or matrix) can be produced by making a vector with vector entries. If we modify the previous example slightly by replacing the expression `n^2` with `[n, n^2]` and then repeating the above we get `[[1, 1], [2, 4], [3, 9]]` which displays as a table with the first column containing the value of the index n and the second column containing the value of the expression n^2. This is a good technique for studying patterns in data. See Figure 0.7 on the next page for some examples.

We have already seen two important applications of vectors in Section 0.10; namely,

- Plotting a vector of 3 or more functions $[f(x), g(x), h(x), \ldots]$ plots each of these functions in order.

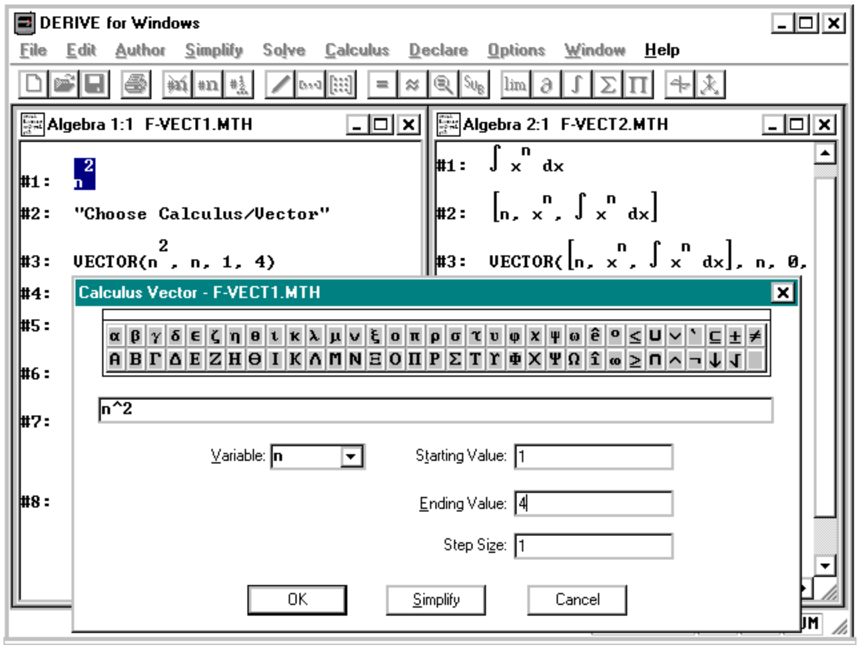


Figure 0.7: Using the Calculus/Vector command

- Plotting a vector of 2-vectors $[[x_1, y_1], [x_2, y_2], \dots]$ will plot the individual points (x_1, y_1), (x_2, y_2), ...

We will have other application that will require us to refer to the individual expression inside of a vector. This is done with the DERIVE `SUB` function (which is short for *subscript*). Thus, for example, `[a,b,c] SUB 2` simplifies to the second element b. DERIVE will display this as $[a, b, c]_2$ which explains the name. For a matrix or vector of vectors then double subscripting is used so that, for example, if

$$y := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

then **A**uthoring `y SUB 2 SUB 1` will be displayed as $y_{2,1}$ and simplify to 3 (because it's on row 2 and column 1).

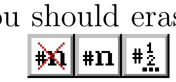
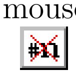
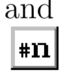
0.15 Printing and Saving to a Disk

You can save the expressions in an algebra window to either a floppy disk or the network hard drive H: and come back later to continue working on them. Unfortunately, the plot windows are not saved but the pictures can be put on the clip board and saved as graphics files using suitable graphics software. To save to a floppy, put a the diskette in say the A: drive and activate the algebra window that you want to save. Click File/Save As and fill out the menu of options with the drive A: and a file name such as Lab5 or just save to A:LAB5 and enter. DERIVE will add the extension .MTH to indicate that this is a file consisting of DERIVE expressions. After the file has been saved you can update it by simply pressing the  button.

You will most likely save your files to the network harddisk. The H: drive (H is for 'Home') is *your private area* which is accessible only using your password. To save a file, just refer to it as say H:LAB5 or switch to the H: drive and view your files.

Later, you can recall these expressions by using either the File/Open or File/Load/Math options. The second method is used primarily to add expressions to an existing window. If you forget the name of your files just type either A: or H: and press the enter–key to see a listing of your files.

When you do a file operation you will notice that the default directory C:\DfW\M206L has lots of files of the form F-*.MTH. These files are the algebra window expressions from the various figures in the manual. For example, Figure 0.7 on the preceding page has two algebra windows F-VECT1.MTH and F-VECT2.MTH which are identified in the top stripe of the corresponding algebra windows. You can load these files at any time to see how expressions are entered or to experiment with the material.

During the course of your session with the computer you will make lots of typing and mathematical mistakes. Before saving your work to a file or before printing and turning your lab in for grading you should erase the unneeded entries and clean up the file. The three buttons  can be used for this purpose. For example, if you select several expressions by say dragging the mouse pointer over them with the left button held down and then press the  button these expression will be removed. Clicking  will undo the last delete. You can move a block of highlights lines by holding down the right mouse button and dragging the block to a new location. Of course, when you delete or move some lines then the line numbers will no longer be

in a proper sequence of #1, #2, ... You can correct this by pressing the renumbering button . You should practice these commands on some scratch work to make certain you understand them.

One way to use the move command is to write comments in the file and placing them before computation. Many of the *.MTH files that we wrote for this lab manual use this technique. To do it, just author a line of text enclosed in double quotes, for example, `"Now substitute x=0."`. Then, move this comment to the appropriate location.

You can print all the expressions in the algebra window (even the ones you can't see) by pressing the button. You do the same thing to print a graph. Just activate the plot window and press . Typically, students turn in the labs by printing out the algebra window and penciling in remarks and simple graphs. More extensive graphs can be printed out. Some combination of hand writing and printouts should be the most efficient.

0.16 Help

You can obtain on-line help by choosing Help. This help feature provides information on all DERIVE functions and symbols. Suppose that you want to know how to enter the second derivative of a functions $f(x)$ by typing. For example, maybe this expression is to be used as part of another function. There are three techniques for learning how to do this.

The first method starts by authoring `F(x):=` to declare $f(x)$ to be a function of x. Next we use the menus with Calculus/Differentiate to calculate the second derivative by entering `F(x)` for the function and 2 for the order. Then, press Author followed by the pull-down key `F3` or right click in the author box and select Insert. This will enter DERIVE's way of typing the expression, in this case it's `DIF(f(x),x,2)`. The second method is to use the online help by choosing Help on the main menu. One then searches for a topic like differentiation or vector to get further information.

0.17 Common Mistakes

Here are a few common mistakes that everyone makes, including the authors, every once in a while. It just takes practice and discipline to avoid these problems, although, it is human nature to blame the computer for *your*

own mistakes. Fortunately, the computer never takes insults personally and it *never* takes revenge by creating sticky keys, erasing files, locking up, or anything else like that ... or does it?

Q1. I tried to plot the line $ax + b$ and instead I got an error message about "too many variables". What did I do wrong? You must define a, b to have numerical values, otherwise DERIVE treats your function as $f(a, b, x)$ which it cannot plot.

Q2. I tried to plot the family of parabolas $x^2 + c$ and I got an error about too many variables. What did I do wrong? Same problem as above, except now DERIVE is trying to plot a surface $z = f(x, c)$in 2-dimensional space. You probably want to enter and Simplify a vector of functions such as

```
VECTOR(x^2 + c, c, 0, 4).
```

Now Plot this vector of 5 functions: x^2, $x^2 + 1$, $x^2 + 2$, $x^2 + 3$, and $x^2 + 4$.

Q3. I entered the expression $\sqrt{5 - x}$ correctly, but when I substituted $x = 9$ and simplified I got $2\hat{\imath}$. What happened? You took the square root of a negative number which is not allowed when you are working with the real number system. DERIVE treats this as a computation with complex numbers and uses the complex number i (where $i^2 = -1$).

Q4. I solved for the 3 roots of the cubic $x^3 - 2x^2 + x - 2$ and I got $x = 2$ which I guessed from the graph but the other two solutions were $x = \hat{\imath}$ and $-\hat{\imath}$. Where do these last two come from? If you factor the cubic instead of using Solve you would get $(x - 2)(x^2 + 1)$. The complex solutions come from that quadratic term. In calculus, we just ignore those complex solutions. For example, numerically solving the above cubic will give only real solutions.

Q5. I differentiated e^x and I got $e^x \ln e$, what's wrong? Nothing, DERIVE is treating the letter e as an ordinary symbol like a or b. You probably wanted Euler's constant e which can be entered using the symbol bar or with #`e`.

Q6. I tried to author the inverse tangent function `arctan x` and I got $a \cdot r \cdot c \cdot \tan x$ instead. What's wrong? DERIVE recognized the `tan x` part but treated the other symbols as individual constants. Use `atan x`.

Q7. I entered the vector $[v_1, v_2, v_3]$ by typing `[v1,v2,v3]` and I got $[v \cdot 1, v \cdot 2, v \cdot 3]$ instead. What happened? You must declare these multi-letter variables first before they can be treated as a single variable. To do this just author `v1:=`, `v2:=` and `v3:=`. A quick way to do this is to simply author the vector `[v1:=, v2:=, v3:=]`.

Q8. I tried to author `x^n` and I got a syntax error! How was this possible? The problem here is that either x or n is previously defined as a function. For example, maybe you had authored $x(t) := \sin t$. You can check on this by scrolling up to find the definition. If instead, you know the problem is that $x(t)$ is defined and you want to remove that definition, then just author `x:=`. In extreme cases you might just open a new window and copy over some of your expressions using the Copy and Paste technique.

Q9. I entered and simplified $\sin(2\pi)$ and I got `SIN([2π])` instead, what happened? You authored `sin[2pi]` instead of `sin(2pi)`. DERIVE treats square brackets not as parenthesis but as a device for defining *vectors*, see Section 0.14.

Q10. I tried to show that $\lim_{n\to\infty}(1 + \frac{1}{n})^n = e$, instead DERIVE returns a question mark indicating that it can't do this problem. What's wrong? Same as above, check that your parenthesis are not brackets.