Derive™ Tutorial Ten

Differential Equations

Introduction to Standard Forms

You can write a first-order differential equation in the form $p(x,y)+q(x,y)y'=0$. For example, $(x^2+4)y'+3xy=6x$ can be rewritten as $3xy-6x+(x^2+4)y'=0$ where $p(x,y)=3xy-6x$ and $q(x,y)=x^2+4$. There is also a standard form for second-order differential equations. Derive uses these forms in the syntax for solving differential equations. Derive can give a general solution or a particular solution to a differential equation.

First-Order Differential Equations

Derive uses the DSOLVE1 command to solve first-order differential equations with $p(x,y)$ and $q(x,y)$ determined from the standard form. The syntax for DSOLVE1 is:

```
DSOLVE1(p(x,y),q(x,y),x,y)
```

where x is the independent variable and y is the dependent variable. For example, the simple first-order differential equation $y'=y$ can be rewritten in the standard form $-y+y'=0$ where $p(x,y)=-y$ and $q(x,y)=1$. You may recognize that a function that is a solution to this differential equation is the function $y(x)=e^x$ since the differential equation states that the derivative of the function is the function itself.

Solve this differential equation by Authoring DSOLVE1(-y,1,x,y) and clicking on Simplify. The $x0$ and $y0$ are the initial values of x and y (and can be consolidated into an arbitrary constant for the general solution). You can solve for y as follows:

```
solve(DSOLVE1(-y,1,x,y),y)
```

You may recall that the solution to this differential equation is $y(x)=Ae^x$ which does not look like the solution Derive gives. With a little mental algebra, you can see that $y_0e^{x-x_0}=y_0e^xe^{-x_0}=Ae^x$ where $A=y_0e^{-x_0}$.

To obtain the particular solution to $y'=y$ through the point (0, 1), Author the following:

```
solve(DSOLVE1(-y,1,x,y,0,1),y)
```

Slope Fields For Linear Differential Equations

To display the slope or direction field for a first-order differential equation $y'=x^2$, Author

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DIRECTION_FIELD(x^2 , x, -3, 3, 12, y, -3, 3, 12),
```

Simplify and click on the 2D-Plot icon in the Main menu. To display the direction field correctly in the Graph window, do the following.

1. Click on Options and select Display
2. In the dialog box, click on Points
3. Click on the Yes button so that solid line segments will be displayed
4. Click OK
5. Reset the Plot Range
6. Click on the 2D-Plot icon in the Graph window.

The direction field will be displayed. The direction field for $y' = x^2$ will be cubic in form as you know from your previous experience in calculus with antiderivatives.

A more interesting direction field occurs for the differential equation $y' = y - x$. To display this equation's direction field, you should work with the following:

DIRECTION_FIELD(y - x, x, -3, 3, 12, y, -3, 3, 12).

You will notice in this direction field that there appears to be one solution to the differential equation that is a straight line. Can you estimate what the equation of this straight line is and check that it is a solution using substitution? Can you use the symbolic solution capability of Derive to solve this differential equation?

Second-Order Differential Equations

Derive can solve second-order differential equations that can be written in the form $y'' + p(x)y' + q(x)y = r(x)$. The syntax for the Derive command is as follows:

DSOLVE2(p(x),q(x),r(x),x,c1,c2)

where c1 and c2 are arbitrary constants. For example, the second-order differential equation $y'' = \sin(x)$ can be rewritten as $y'' + 0y' + 0y = \sin(x)$ where $p(x) = 0$, $q(x) = 0$, and $r(x) = \sin(x)$. Thus the Derive command is

DSOLVE2(0,0,sin(x),x,c1,c2).

Author and Simplify this command to display the solution to the differential equation.

Notice that with DSOLVE1 the result is an equation from which you can sometimes get an explicit solution whereas with DSOLVE2 the result is an expression for the solution. Initial-value differential equations can be solved with DSOLVE2_IVP and boundary-value differential equations can be solved with DSOLVE2_BV. The syntax for these commands can be found by clicking on Help in the Main menu and selecting Online.

