Derive™ Tutorial Eleven

Matrices

Systems of Linear Equations

You can use the Solve command to solve systems of equations. For example, to solve

$$\begin{cases} 2x+3y-4z = -3 \\ x-y-6z = 2 \\ 3x+2y-z = 5 \end{cases}$$

Author

`SOLVE(2x + 3y - 4z = -3 and x - y - 6z = 2 and 3x + 2y - z = 5, [x, y, z])`

and Simplify. The solutions are displayed as rational numbers. However, you can use arrays of numbers or matrices to more easily enter and solve such systems.

You can represent this system of equations with matrices by concentrating on the coefficients. For example,

$$\begin{cases} 2x+3y-4z = -3 \\ x-y-6z = 2 \\ 3x+2y-z = 5 \end{cases}$$

can be represented by

$$\begin{bmatrix} 2 & 3 & -4 & -3 \\ 1 & -1 & -6 & 2 \\ 3 & 2 & -1 & 5 \end{bmatrix}$$

where you can see that the first column is the coefficients of the *x*-terms, the second column the *y*-terms, the third column the *z*-terms, and the fourth column is the constant terms. This array of numbers is called a 3 by 4 matrix (often written 3×4 matrix). You can solve such systems of linear equations represented by the corresponding matrix using matrix operations. To do this, you must first enter this matrix. First use the Author>Matrix command and in the dialog box, enter 3 for the number of rows, 4 for the number of columns and click on OK. Now enter the elements of this matrix pressing the Tab key after each entry. Notice that the fourth column appears when you tab after the third entry. Click on OK when you are finished.

To solve this system of equations, Author `Row_Reduce`(*#n*), where n is the expression number of the matrix , and Simplify. When you convert the solution matrix

$$\begin{bmatrix} 1 & 0 & 0 & \frac{53}{15} \\ 0 & 1 & 0 & -\frac{37}{15} \\ 0 & 0 & 1 & \frac{2}{3} \end{bmatrix}$$ into equation form, you obtain $$\begin{aligned} 1x+0y+0z &= \frac{53}{15} \\ 0x+1y+0z &= -\frac{37}{15} \\ 0x+0y+1z &= \frac{2}{3} \end{aligned}$$

From this system of equations, you can see that the solutions are the same as you obtained with the Solve command. However, this matrix method allows you to enter just the coefficients and constants. For large systems of linear equations, the matrix method saves time.

Matrix Multiplication

You can represent this system of equations using matrix multiplication.

$$AX = B$$

where

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 1 & -1 & -6 \\ 3 & 2 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \text{and } B = \begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix}.$$

The solution to $AX = B$ is obtained by multiplying both sides on the left by the inverse of $A\left(A^{-1}\right)$. Thus, $X = A^{-1}B$. To obtain the solution, enter the coefficient matrix A by Authoring

```
A:=[2, 3, -4; 1, -1, -6; 3, 2, -1],
```

and enter the constant matrix B by Authoring

```
B:=[-3; 2; 5].
```

Finally, Author `A^(-1)*B` and Simplify. The solution is displayed as

$$\begin{bmatrix} \frac{53}{15} \\ -\frac{37}{15} \\ \frac{2}{3} \end{bmatrix}.$$

Thus the solution is $x = \frac{53}{15}$, $y = -\frac{37}{15}$, and $z = \frac{2}{3}$ as before. You can check your result by multiplying A by the solution matrix as

$$\begin{bmatrix} 2 & 3 & -4 \\ 1 & -1 & -6 \\ 3 & 2 & -1 \end{bmatrix} * \begin{bmatrix} \frac{53}{15} \\ -\frac{37}{15} \\ \frac{2}{3} \end{bmatrix}.$$

The result should be the constant matrix B.

Other Matrix Commands

There are many other matrix commands. Some of the more frequently used are:

determinant	DET
trace	TRACE
transpose	`
rank	RANK

Each of these is a function with the syntax

FUNCTION(*matrix*)

except for transpose which follows the matrix as did the inverse operator.