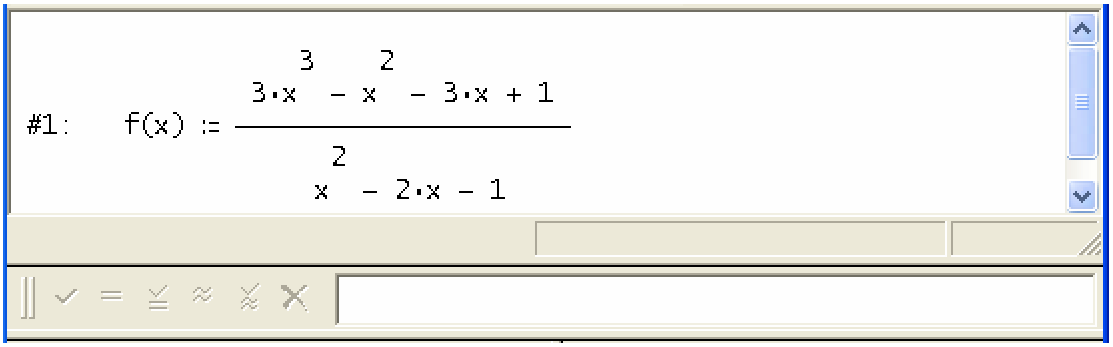
Derive™ Tutorial Three

Investigating a Rational Function

In this lesson, you will examine in detail the graph and the behavior of a rational function. In so doing, you will demonstrate Derive features that are related to factoring, extracting parts of expressions, limits, establishing graphing windows, and graphing. Return to the Algebra window, if necessary. You will begin with the following function:

$$f(x) = \frac{3x^3 - x^2 - 3x + 1}{x^2 - 2x - 1}$$

Define a function using the Author>Function command. Enter the name of the function along with the argument x as $f(x)$. Press the Tab key. Enter the definition of the function as (3x^3-x^2-3x+1)/(x^2-2x-1). Now click on OK. You should see the following.



You would now like to factor the numerator and denominator to determine the zeros and domain of the function. Click on the fraction bar in $f(x)$. Then click on the numerator (top) of the rational expression. With this numerator now highlighted, use the Simplify>Factor command and select Factor. On the right of the displayed window, click on the Rational polynomial button. Now click on Factor in the dialog box. You will see the rational function with a numerator that is factored.

Now for the denominator. Click on the fraction bar of $f(x)$ again and then on the denominator. Now use the Simplify>Factor command again. Notice that the Rational polynomial button is still on. Click on Factor in the dialog box.

This time the denominator is not factored because it does not have rational factors. Highlight the denominator by clicking on the fraction bar and then on the denominator. Use Simplify>Factor again and this time click on the Radical expression button.. Now press Factor. Notice that the denominator is now factored with radicals.

You can name the zeros of the function simply by deciding which numbers make each of the polynomial numerator factors zero. These numbers are 1, -1 and 1/3.

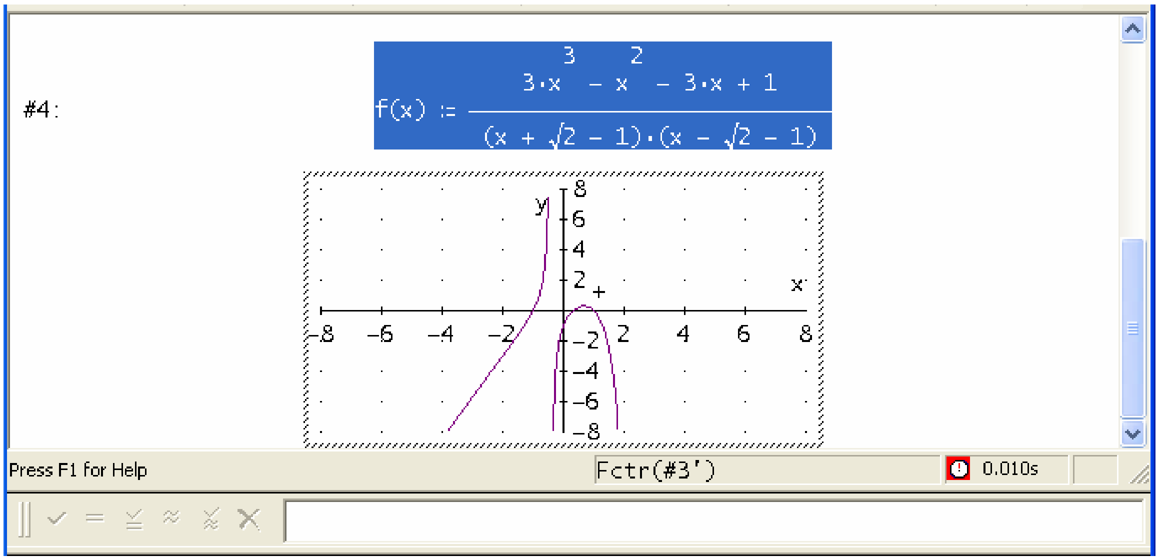
To find the real numbers that are not part of the domain, you need to find the zeros of the denominator. This is a bit more difficult. But you observe that the first factor is zero when x is $1-\sqrt{2}$ and the second factor is zero when x is $1+\sqrt{2}$.

Here you are using the power of Derive to factor the polynomials and then your own understanding of polynomials to find the zeros of these factored polynomials.

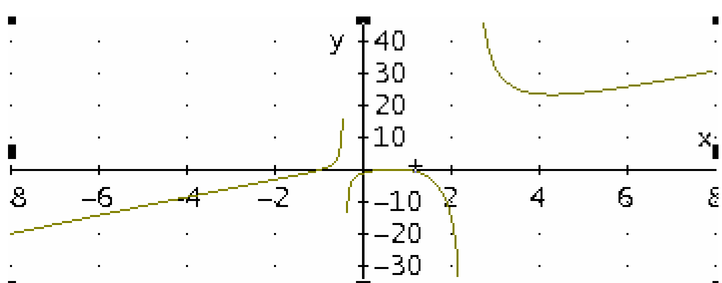
Now for the graph of the function. With the factored function highlighted (as it should be now), click on the 2D-Plot icon that is the third from the last icon on the Icon menu bar. You are now at the graphing

window and you thought that the graph of the function would be displayed. However, you have to click on the 2D-Plot icon that is now to the left of the middle of the screen to cause the graph of your highlighted function (back on the algebra screen) to be displayed. Delete old graph, if necessary. Do that now.

You will see a graph that has zeros at -1, 1, and 1/3 as you determined earlier. To insert this graph in the Algebra window, use the File>Embed command. Now click on the algebra icon to far right of the icon bar. You should see the graph (as shown below) inserted in the Algebra window. You can easily see that the zeros and omitted values are correct in the Algebra window.



You can see from the graph that there might be a vertical asymptote at $-1-\sqrt{2}$ as you suspected from your algebraic work. Now rescale the graph so that you can see the position of the vertical asymptote at $-1+\sqrt{2}$. To do this, highlight the function, click on the graphing window icon, and then, use the Algebra>Options command. In the Options menu, select AutoScale New Plots. Now click 2D-Plot. Your plot should now look like this.



In this new graphing window, you see that there is a vertical asymptote at about $-1+\sqrt{2}$.

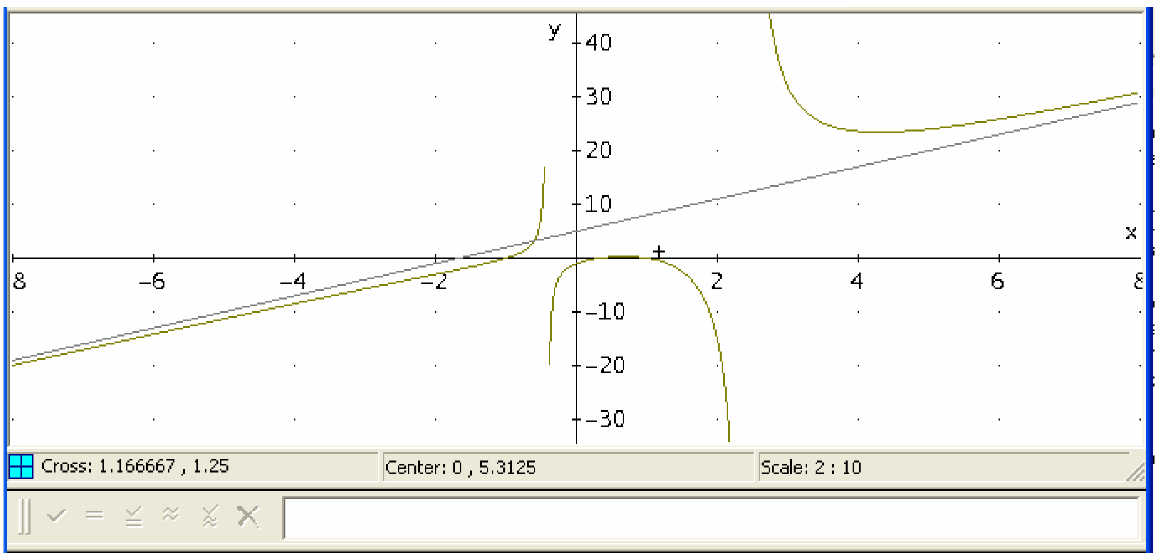
Zoom out once by clicking on the icon with the four outward pointing arrows. The graph seems to reveal an oblique asymptote with a positive slope. To determine what this oblique asymptote might be, you can divide the numerator by the denominator to get a polynomial plus a smaller rational function. To do this, go to the Algebra window and highlight the rational expression for the function (do not include f(x) itself). Now in use the Simplify>Expand command and then click on the Expand button with the Rational polynomial button turned on. Your expression should now look like this.

$$f(x) := \frac{4\cdot\sqrt{2} + 5}{x - \sqrt{2} - 1} + \frac{5 - 4\cdot\sqrt{2}}{x + \sqrt{2} - 1} + 3\cdot x + 5$$

You can see that the two rational expressions both become small when x gets large in absolute value. Thus, the rational function $f(x)$ gets close to the linear function $3x+5$ as x gets large in absolute value.

You can check this by taking the limit of the two rational expressions in this representation of $f(x)$ as x goes to infinity. You do this by highlighting this latest function expression without the f(x):=, by clicking on the fraction bar. Now copy this highlighted expression by pressing Ctrl-c. Clear the Entry line, if necessary, and press Ctrl-v to paste the expression into the Entry line. Delete the linear expression (3x+5) and the plus sign preceding it. Now press Enter to Author this line and then click on the Calculus menu item. Select Limit on this pop-down menu and then type inf (for infinity) in the Limit point box replacing whatever appears there. Finally, click on the Simplify box at the bottom of this window. You can see that the limit of the rational expressions is zero. Thus, the oblique asymptote is y=3x+5.

You can graph this asymptote by Authoring 3x+5 and then clicking on the 2D-Plot icon and then on the 2D-Plot icon in the graphing window. Your graph should now look like this.



Notice that this asymptote is below the graph to the right of the y-axis and above the graph to the left of the y-axis. Notice also that the oblique asymptote intersects the graph to the left of the y-axis. You may find this unusual. However, it is correct and you should use your mathematical knowledge to figure out why it is correct.