Derive™ Tutorial Five

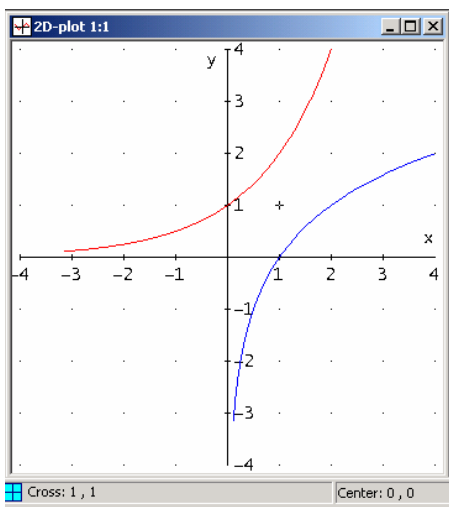
Exponential and Logarithmic Functions

You can graph exponential functions to show the effect of changing the base of the exponential functon. To do this, first Author the expression 2^x. Click on the 2D-Plot icon, delete all plots and Slider Bars, reset the Plot Range, and turn off AutoScale New Plots under Options, if necessary. Then click on the 2D-Plot icon to display the graph of the exponential function defined by this expression. Notice this is an increasing function that passes through the point (0, 1) and is asymptotic to the *x*-axis as $x \to -\infty$.

Now Author 2^(-x) in the Graph window and display its graph. Notice this is a decreasing function that passes through the point (0, 1) and is asymptotic to the x-axis as $x \to +\infty$. In the Graph window, use the command Edit>Delete All Plots.

Now you will use parametric graphing to graph the inverse of the exponential function $f(x) = 2^x$. Square brackets are used for entering the formulas for the first and second coordinates of a point on the graph of the function. To graph $f(x) = 2^x$ in this parametric form, delete all plots, Author [t, 2^t] and display the graph (if the Parametric Plot Parameters dialog box appears, click on OK). In the Graph window, use the Set>Aspect Ratio command. Then change the Horizontal value to match the Vertical value and click on OK. The graph is now displayed in a smaller window with the same *x*-axis and *y*-axis scaling.

To graph the inverse of $f(x)$, you reverse the coordinates. Thus, you Author [2^t, t] and display its graph. The graph should look something like this.



Now click on the Trace icon and observe the coordinates of the point where the cursor is located. Press the up arrow on the keyboard to move to the other graph. Pay close attention to the coordinates displayed in the lower left of the window and the value of *t* at the top of this Graph window. Notice that if the value of t is the same for both graphs, then the coordinates switch as you move from one graph to the other. You should check this switching with other points. A function and its inverse have coordinates that are switched. The inverse of the exponential function base 2 is called the logarithm function base 2. The standard notation for this inverse function is $f^{-1}(x) = \log_2(x)$.

In the Graph window, use the command Edit>Delete All Plots.

To see how the base changes both the exponential and logarithmic functions, you can use animation. To do this, Author [t,a^t]. Before clicking on the Graph icon, use the Insert>Slider Bar command and in the dialog box that appears, type a for the Variable Name, 2 for the Minimum Value, and 20 for the Intervals. Then click on the Update box and click on OK. Now click on the Graph icon. Move the arrow in the

displayed Slider Bar to the right to see how the exponential function graph changes as the base increases from 2 to 10.

Without deleting the graph of the exponential function, Author the logarithmic function by entering [a^t, t] and pressing Enter. In the Graph window, click on the Graph icon. Now move the Slider Bar to see how both the exponential and logarithmic functions change as the base changes. As the base increases, the exponential function gets steeper while the logarithmic function flattens out. Observe also that the paired functions are symmetric to the line $y = x$.

Converting from Logarithmic Form to Exponential Form

Return to the Algebra window. Author log(8,2). Use the Simplify>Basic command to simplify the expression. The Derive expression log(8,2) is equivalent to the standard mathematical expression $\log_2(8)$ whose value is 3. Now Author y=log(x,2) and then solve this equation for x using the Solve >Expression in the Algebra Window's Main menu. The result is $x = 2^y$. Here, you have that $y = \log_2(x)$ is equivalent to $x = 2^y$. Thus, you can switch between a logarithmic expression and its equivalent exponential expression. You have seen earlier that the logarithmic function and the exponential function with the same base are inverse functions of one another.

You can convert expressions of the form log(x,base) to a decimal approximation using the command Simplify>Approximate.

Solving Exponential and Logarithmic Equations

You can solve exponential equations like $2^{x+1} = 5$. To do this, Author 2^(x+1)=5. Solve for x using Solve >Expression on the Algebra Window's Main menu. Remember to select Algebraically when solving. The result is $x = \frac{\ln(5)}{\ln(2)} - 1$. This is an exact solution for x. You can solve the original equation approximately by choosing Numerically in the Solve>Expression dialog box. More complicated exponential equations can be solved in the same way.

Logarithmic equations can also be solved with Derive. To solve the logarithmic equation $\log_3(x) + \log_3(x-1) = -2$, you begin by Authoring log(x,3)+log(x-1,3)=-2. Solve this equation algebraically . The solution $x = \frac{\sqrt{13}}{6} + \frac{1}{2}$ looks like it might be the solution to a quadratic equation. You might think about why this is.

