Derive™ Tutorial Seven

Derivatives

The slope of the tangent line to the graph of the function $f(x)$ at a point $(x, f(x))$ on its graph is

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$

You can begin to find the value of this expression for the function $f(x)=x^2$ by Authoring this function. To do this, use the Author>Function Definition command. In the dialog box that appears, enter f(x) on the first line and press the Tab key. In the large Function Definition box, type in the expression x^2. Click on OK. You can now simplify the expression

$$\frac{f(x+h)-f(x)}{h}$$

by Authoring it in the Entry line as (f(x+h)-f(x))/h. To see the step-by-step simplification of this expression with $f(x)=x^2$, use the Simplify>Display Step command. The first simplification has two terms in the numerator. You can finish the simplification by using this process again. The expression that results, 2x+h, will approach 2x as h approaches 0. You can perform this entire process by Authoring

(f(x+h)-f(x))/h.

Use the Calculus>Limit command and in the dialog box that appears, type *h* in place of *x* and 0 for the Limit point and then click on OK. This places the highlighted full limit expression in the History window. In the Icon menu bar, click on = to give the value of the limit (which is 2*x*).

Since the slope of the tangent line to the graph of a function is the derivative, you can write

$$\frac{df(x)}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

where $\frac{df(x)}{dx}$ is the derivative of $f(x)$ with respect to *x*. You can find the derivative of your function using the Calculus menu. Author f(x) and then use the Calculus>Differentiate>Simplify command. The derivative is, as you know, 2*x*.

You can compute the derivative of any function that can be defined by an expression (if the derivative exists) using only the expression. For example, the derivative of $g(x)=\frac{3x^2-5x+2}{5x^4-3x+2}$ can be found

by Authoring the expression dif((3x^2-5x+2)/(5x^4-3x+2),x) and Simplifying, where dif is an alternate form of the derivative operator and you must indicate the variable of differentiation. The backward 6 (∂) to the right of the Greek alphabet among the mathematical constants (like π and *e*) and operators at the bottom of the screen can be used instead of dif.

Taylor Series

You can represent a function with an extended polynomial or Taylor series. To do this, Author the expression *e*^(-x^2) (be sure to click on the *e* in the lower right portion of the screen), then use the Calculus>Taylor Series command. In the dialog box that appears, enter 0 for the Expansion Point, 2 for the Order and then click Simplify. A polynomial of degree 2 (Order 2) is displayed. To obtain a polynomial approximation of degree 4 for e^{-x^2}, highlight e^{-x^2}, use the Calculus>Taylor Series command and enter 4

for the Order. Then click on Simplify as before. A polynomial of degree 4 appears.

Continue this process for Orders 6 and 8.

To see that these Taylor polynomials are approximating polynomials for e^{-x^2}, highlight this expression (or Author it again). Click on the 2D-Plot icon at the right of the Icon menu bar and then on the 2D-Plot icon in the middle of the Icon menu in the Graph window that appears. Delete all plots, if necessary. Go back to the Algebra window by clicking on the right most icon on the Icon menu bar. Highlight the Taylor polynomial of Order 2. Move to the Graph window and graph the highlighted polynomial. Notice that it is a good approximation for the graph of e^{-x^2} on the interval near $[-1,1]$. You can graph the other Taylor polynomials to see that they are better and better approximations to the graph of e^{-x^2}. Remember that you have to click on the polynomial you wish to plot in the Algebra window before you return to the Graph window to plot it. Notice that the interval for which the approximating polynomial is close to the original graph gets wider as the Order or degree of the Taylor polynomial increases.

