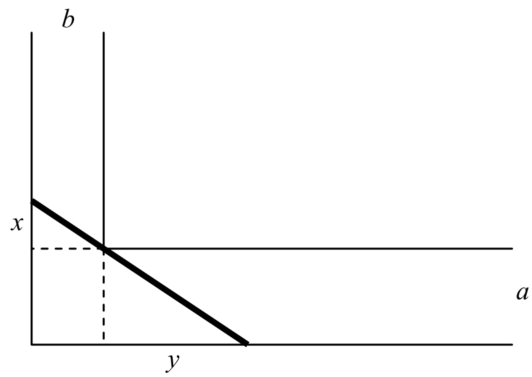
Derive™ Tutorial Eight

Ladder Problem

If you have ever moved furniture, you know it is difficult to move things around corners in a hallway. The figure below illustrates a ladder stuck in a hallway that you wish to move around a corner keeping it level. How long a ladder can you move around a corner in a hallway whose widths are *a* and *b* as in the figure?



You can use the variables *x* and *y* in the diagram in representing the length of the ladder using the two similar small right triangles in the diagram. The length of the ladder is $\sqrt{(x+a)^2+(y+b)^2}$. You would like this equation in one variable, either *x* or *y*. In Derive, Author the expression in *x* and *y* for the length of the ladder using the square root symbol on the lower right of your screen, namely,

√ ((x+a)^2+(y+b)^2)

From the two small similar right triangles, you can see that $\frac{y}{a}=\frac{b}{x}$ or $y=\frac{ab}{x}$. You can create an expression for the length of the ladder that involves only the variable *x*. To do this, use the command Simplify>Variable Substitution. In the dialog box displayed, select *y,* type a*b/x for New Value , and click on Simplify.

Now create a function for the length of the ladder with this expression. To do this, copy the highlighted expression using the Edit>Copy command. Then use the Author>Function Definition command and in the dialog box that appears, type in ladder(x), press the Tab key, and press Ctrl-v to paste in the expression that you copied and click on OK. You would like to find the longest ladder that can be moved around the corner. So, you try to find the zeros of the derivative. To do this, first find the derivative operator symbol (∂) among the mathematics operators at the lower right of the screen. Author ∂(ladder(x),x)=0. Notice that the expression $\frac{d}{dx}\text{ladder}(x)=0$ is in standard derivative form. Now use the Solve>Expression command. Be sure that the variable *x* is highlighted and Algebraically and Real are selected in the dialog box. Then click on Solve. The result $x=a^{1/3}b^{2/3}$ is a positive number because *a* and *b* are positive.

You seemed to have completed the problem. You might want to check the second derivative to determine if this zero of the derivative gives a maximum or a minimum. To do so,

Author ∂(ladder(x),x,2).

Notice the 2 at the end of the Authored expression. When you simplify this expression, you get a complicated expression. Because *a*, *b*, and *x* are positive, this expression is positive including SIGN(x+a) which is the number 1 if *x*+*a* is positive. From your calculus course, you know this indicates that a minimum exists at the only critical point. How can this be?

Graph the ladder function in the Graph window by highlighting the expression for the ladder function. To

set a=8 and b=27, use the Simplify>Variable Substitution command. In the dialog box, highlight a and enter 8 in the New Value box. Do the same thing for b and click on OK in the dialog box and Simplify the result. Click on the 2D-Plot icon and Set the Plot Range to [0,50] x [0,100]. The graph is the graph of the lengths of the ladder that don't fit around the corner. Thus, the minimum ladder length that does *not* fit around the corner is the maximum length that does (in a perfect world).

