Derive™ Tutorial Nine

Integration

Indefinite Integrals

You can integrate a variety of simple and complicated functions using Derive. Many one variable functions, however, that look simple have no closed form antiderivatives (indefinite integrals). The definite integral of some functions that are discontinuous are often called improper and can exist as real numbers. Finally, functions of several variables can be integrated with Derive with the previous cautions.

You can find the indefinite integral of the function $f(x) = x^2$ by Authoring `x^2`, and using the command Calculus>Integrate. In the dialog box, make sure that the Indefinite button is on and click on Simplify. You will notice that the antiderivative is correct, but the indefinite integral is a set of functions that needs an arbitrary constant. Press Enter to Author `x^2` again. Use Calculus>Integrate again and in the dialog box, change 0 to c for the constant in the lower right corner. Now click on Simplify to display the correct indefinite integral with a constant of integration.

Notice the syntax in the last integral on the screen. Author this integral (using the symbol palette at the lower right of the Derive screen to enter the integral sign). Notice that the result is the same. If you change the integral sign to `int`, so that the Entry line looks like this:

```
int(x^2,x,c)
```

and click on the check mark with the equal sign underneath it (the same as Enter>Simplify) at the lower left of the Entry line, the same result is displayed again. You can see that there are various ways of entering an indefinite integral. You should choose one that is easy for you to remember.

Derive can integrate all indefinite integrals that appear in any published table of integrals. For example, you can integrate more complicated functions that involve products and quotients. Integrate each of the following using your favorite method of Authoring integrals:

$$\int x\sin(x)\,dx$$

$$\int \frac{x+1}{x^2-3x-10}\,dx$$

$$\int \frac{(x+1)^2}{x^2-3x-10}\,dx$$

$$\int \sec^8(x)dx$$

Definite Integrals

Definite integrals are just as easy to compute. To find $\int_1^5 x^2dx$, Author `int(x^2,x,1,5)`, and Simplify.

The result, 124/3, is immediate.

Derive can compute the exact value of the definite integral of a function on an interval where the function is continuous and has an antiderivative. Derive can compute an approximate value for definite integrals that do not have an antiderivative on any interval where the function is continuous. For example, Author int(cos(x)/x,x,1,3), click on check-equal. The result shows that an antiderivative does not exist for this function . To get an approximation for this definite integral, use the Simplify>Approximate command, and in the dialog box, click on Approximate. An approximate value for this definite integral is displayed. You can use this technique to approximate the value of a definite integral when its exact value cannot be found.

Improper Integrals

Definite integrals of functions which are not continuous on the interval of integration are called improper integrals. Values for such improper integrals are computed using limits. For example, the function in the integrand of the definite integral $\int_0^1 \frac{1}{\sqrt{x}}\,dx$ is discontinuous at the lower limit of the interval of integration.

Thus, it is an improper integral. Its value is computed using $\lim_{a\to 0^+} \int_a^1 \frac{1}{\sqrt{x}}\,dx$ if it exists. To attempt to compute this value, Author lim(int(1/√x,x,a,1),a,0,1) and Simplify. This improper definite integral has an exact value of 2.

Definite integrals whose upper or lower limits are infinite are also called improper integrals. Limits are again involved in computing their values if they exist. You can attempt to evaluate the improper integral $\int_1^\infty \frac{1}{x^2}dx$ by Authoring lim(int(1/x^2,x,1,b),b, ∞) (the symbol for infinity is located on the lower right of your screen) . The exact value of this improper integral is 1.

Approximating Definite Integrals

You can attempt to evaluate the definite integral $\int_1^3 \frac{\cos(x)}{x}\,dx$ by Authoring int(cos(x)/x,x,1,3) and simplifying. Notice that the definite integral is returned without evaluation. To approximate this definite integral, use the Simplify>Approximate command and in the dialog box, click on Approximate. The negative approximate value is displayed.

Errors Evaluating Certain Improper Integrals

All software products have limitations. Derive can give incorrect results for improper integrals of functions whose discontinuity is not at an endpoint of the interval of integration. For example, $\int_{-1}^1 \frac{1}{x}\,dx$ is undefined since there is a discontinuity at 0 and neither of the two limits $\lim_{b\to 0^-} \int_{-1}^b \frac{1}{x}\,dx + \lim_{a\to 0^+} \int_a^1 \frac{1}{x}\,dx$ used to define $\int_{-1}^1 \frac{1}{x}\,dx$ exists. To see Derive's result for this improper integral, Author int(1/x,x,-1,1) and Simplify.

Notice that the value given by Derive is 0 even though you know that the improper integral is undefined. The theory of improper integrals requires that you always check for discontinuties in evaluating definite integrals by hand or with Derive. Recalling that the derivative of $\tan(x)$ is $\sec^2(x)$, you might wish to

investigate the value of $\int_1^2 \sec^2(x)dx$ by hand and then using Derive.

