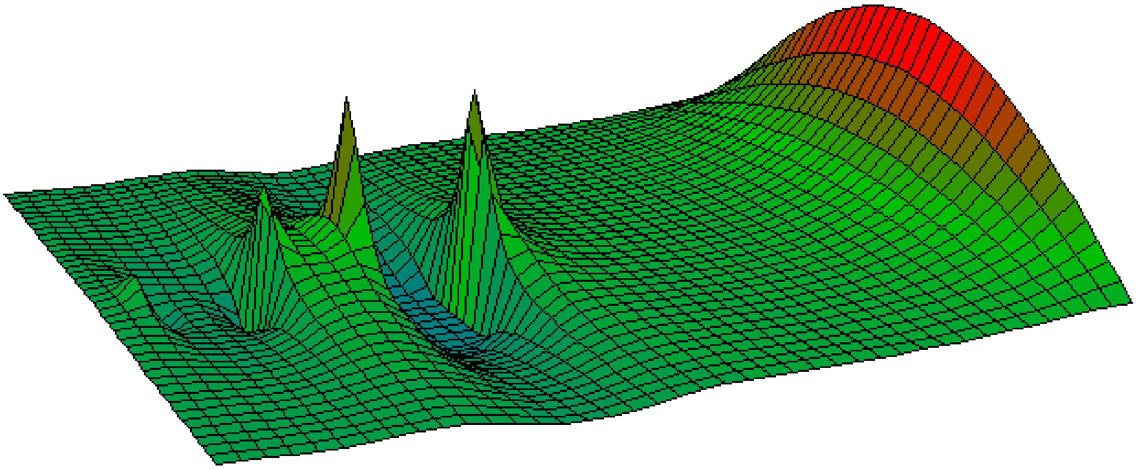
Journal of J

An interdisciplinary journal on J programming language and applications in science



```
r=: 1 2 3 4
   z=: r %:/ y
   z
0 1       2       3       4       5       6
0 1 1.41421 1.73205       2 2.23607 2.44949
0 1 1.25992 1.44225  1.5874 1.70998 1.81712
0 1 1.18921 1.31607 1.41421 1.49535 1.56508
```

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Journal of J (J2-team)

J, a language for the science

Aims and scope

Journal of J is a **non professional and nonprofit** journal in science, involving a large research and users community in J programing language.

Journal of J is an interdisciplinary **journal** devoted to J and science.

Journal of J aims to provide fast access to papers about science and J.

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Style and Contents

Journal of J aims to cover all the main areas of science. Inevitably, articles in different areas are addressed at different audiences. Many of the articles submitted to the journal are standard technical pieces, addressed to a purely academic audience

To attract this variety of contributions ***Journal of J*** will contain the following areas:

- Mathematics: number theory, logic, calculus, algebra, arithmetic, algorithms and others…
- Physics: dynamical systems, chaos, fractals, disorder, statistical physics and others…
- Computer science, Visualization, Computer Art, …

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FOCUS ISSUE (To appear in December - 2011)

Special number about mathematical roots of J. Authors are invited to submit papers for this special number.

Editorial

En este año que casi termina, comienza su andadura este proyecto. *Journal of J* nace como un experimento a raíz de la participación de los editores en el foro de Jsoftware. Como "sucesor" de APL, J es un lenguaje de programación que resulta extraño a primera vista. Sin embargo, tras su inicial dificultad se esconde una potencia y eficacia poco habituales. J es algo más que un lenguaje de programación, es una filosofía. No obstante J requiere cierto tiempo para habituarse. Efectivamente, los programas en J son a veces tan concisos como indescifrables para el no iniciado, y es precisa una cierta dedicación para entender las soluciones propuestas.

Confiamos en que esta revista sirva como punto de encuentro de todos los entusiastas en J. De igual manera esperamos que las interesantes contribuciones de los programadores sirvan como lección y acicate para extender este lenguaje de programación por estos lares.

Mikel Paternain

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Periodicity

Six issues per year

Note:
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"Critique" of the Mathematical Abilities of J

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In this short note, we will review some of the issues raised by Michael Wester in their work <u>*Critique of The Mathematical Abilities of CA Systems*[i].</u>

Of course, J is not a Computer Algebra System, but is a interesting exercise check J in this context. We focus on few aspects, but we suggest exploring the rest.

Symbols

In Wester´s paper a special notation is used to classify the results. In this summary we will use Wester's notation with some changes.

- • success! (hurrah!)
- ★ success but a little fudging or subtlety required, or the answer could be just a little nicer or more complete (yes!)
- •✍ success but you need to write a verb (uff!)

<u>**. Numerical questions**</u>

✓ **C1 Problem •**

$$50! \Rightarrow \mathbf{3.0414093201..0000000000.10^{64}}$$

Big factorials are easy with J and concrete precision,

```
!50
3.04141e64
```

If we need more resolution, we can use

```
  !50x
3041409320171337804361260816606476884437764156896051200000
0000000
```

Extended precision integer constants can be entered as a sequence of decimal digits terminated by an x.

[1] Originally published in *Computer Algebra System: a Practical Guide edited by J. Wester, John Wiley & Sons, Chichester, UK.*

✓ **C2 Problem •★**

$$factor(50!) \Rightarrow 2^{47}3^{22}5^{12}7^{8}11^{4}13^{3}\dots 47$$

In J we have the verb `q:` to obtain prime factors. The answer is correct but need simplification!

```
q: !50x
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3
3 3 3 3 3 3 3 3 3 3 3 3 3 5 5 5 5 5 5 5 5 5 5 5 5 7 7 7
7 7 7 7 7 11 11 11 11 13 13 13 17 17 19 19 23 23 29 31
37 41 43 47
```

✓

✓ **C3 Problem •✍**

$$10!! \Rightarrow 3840, 9!! \Rightarrow 945$$

Some systems lack this feature but in J is possible to define a verb for double factorial (see Essays/Double Factorial – J Wiki)

```
Fe=: 2&^ * !
Fo=: !@+: % 2&^ * !

Fd=: (Fe@-:)`(Fo@-:@>:)@.(2&|)"0
Fd 10
     3840
```

✓ **C4, C5 & C6 Problems**

$$ABC_{16} \Rightarrow 2748_{10}$$

✓ **C7 Problem •**

$$log_8 32768 \Rightarrow 5$$

In J the *base-x logarithm* is x^.y

```
8^.32768
5
```

✓ **C10 Problem •**

$$\frac{1}{2} + \dots + \frac{1}{10} \Rightarrow \frac{4861}{2520}$$

```
1r2+1r3+1r4+1r5+1r6+1r7+1r8+1r9+1r10
4861r2520
```

✓ **C11 Problem •**

$$N(\frac{1}{7}) \Rightarrow \overline{0.142857}$$

```
1%7
0.142857
```

✓ **C12 Problem •**

$$\mathrm{N}\left(\frac{7}{11}\right)\mathrm{N}(\frac{22}{7}) \Rightarrow 2$$

```
(7%11)*(22%7)
```

```
2
7r11*22r7
2
```

✓ **C23 Problem •**

$$2\infty - 3 \Rightarrow \infty$$

```
(2*_)-3
```

```
_
```

✓ **D4 Problem •**

$$\left\lfloor -\frac{5}{3} \right\rfloor \Leftrightarrow -2 \; ; \left\lceil -\frac{5}{3} \Rightarrow -1$$

Floor and Ceiling functions are implemented in J:

```
<.(-5r3)

_2

>.(-5r3)

_1
```

✓ **F9 Problem •✍**

$$\phi(1776) \Rightarrow 576$$

```
totient=: (- ~:)&.q:

totient 1776

576
```

SummaryIn this simple and little exercise, we have seen the good numerical abilities of J. Many issues addressed by Wester remain to be explored in J context.

J-SYMBOL an exercise suggested by the editor

A lo largo de estas páginas podemos encontrar el símbolo superior. Nos encontramos ante una sugerencia y/o ejercicio propuesto por el editor que pretender abrir nuevos campos de empleo de J como herramienta de análisis científico.

Short Note (J2-Team, Mikel Paternain, Editor)

Cantor function

and some experiments.

In J is easy compute the Cantor function:

```
Cantorf =: 3 #. #:
plot Cantorf i.1000
```

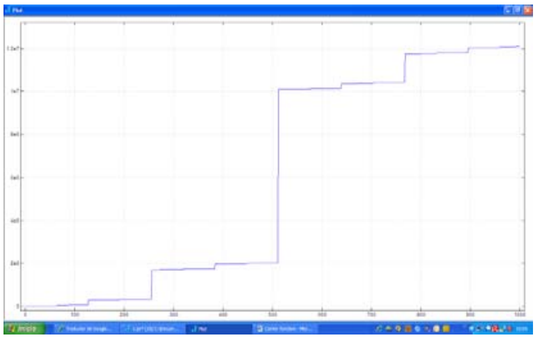
```
Cantorf i.20
```

```
0 1 3 4 9 10 12 13 27 28 30 31 36 37 39 40 81 82 84 85
```

This is A005836-OEIS sequence of numbers whose base 3 representation contains no 2.

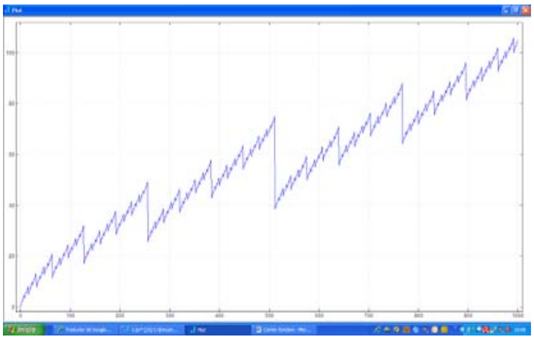
The first natural experiment is change the *radix* and use other number. For example 6:

```
Cantorf =: 6 #. #:
```



Another natural extension is consider a rational number

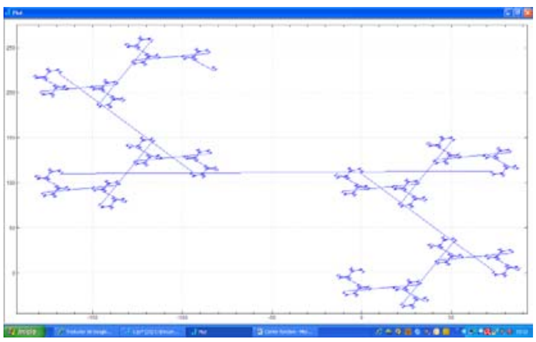
```
Cantorf =: 3r2 #. #:
```



In this case, the change is evident.

Now, we will try a more drastic change:

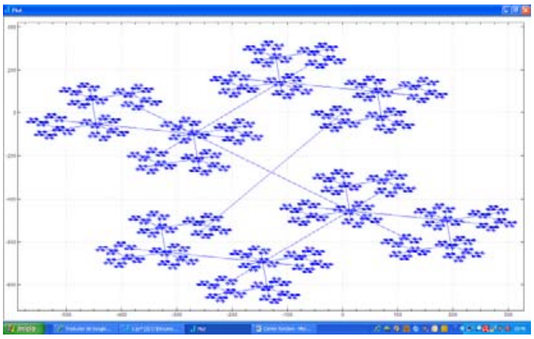
```
Cantorf =: 1j3r2 #. #:

plot Cantorf i.1000
```



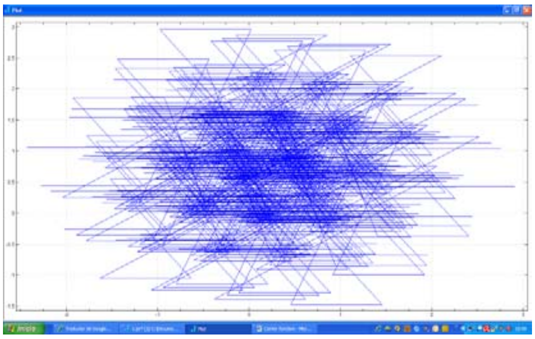
With complex numbers `Cantorf` function lies in the ccomplex domain and we obtain a bi-dimensional graph.

Variying the value of radix, we get a variety of graphics.

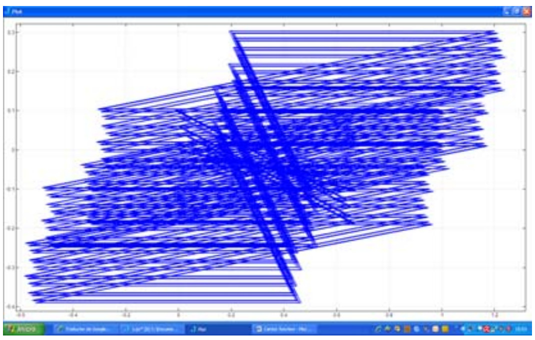
```
Cantorf =: 1r2j3r2 #. #:

plot Cantorf i:30000
```



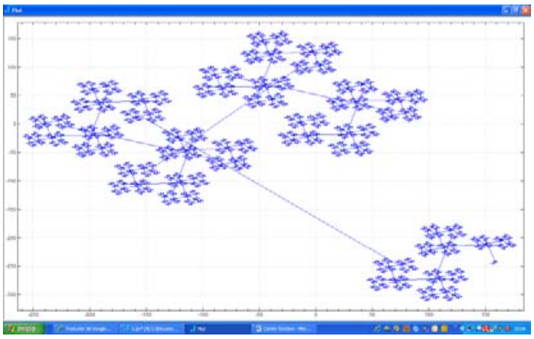
```
Cantorf =: 1r2j4r5 #. #:

plot Cantorf i:1000
```



```
Cantorf =: _4r8j_1r5 #. #:

plot Cantorf i:1000
```



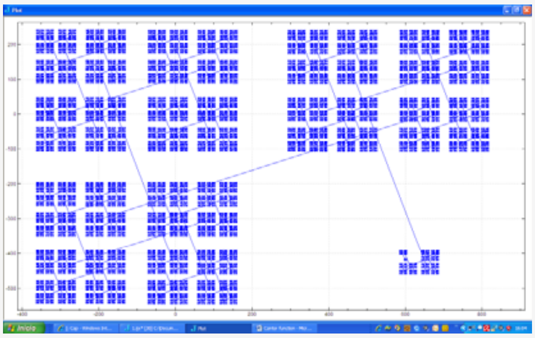
```
Cantorf =: 1r3j3r2 #. #:

plot Cantorf i.10000
```



These graph are similar to the Dragon Curves.

The last example is a fractal carpet.

```
Cantorf =: 1r8000j3r2 #. #:pplot Cantorf i.100000
```



Digital Binary Sums

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We begin with a binary representation of a number. Next, count the number of 1s in this binary representation. This value is $s(n)$. We are interesting about sums $S(n) = \sum_{s=0}^{n} s(n)$.

It is easy to check the values of the following table for $S(n)$.

n	$binary\ base$	$s(n)$	$S(n)$
0	0000	0	-
1	0001	1	1
2	0010	1	2
3	0011	2	4
4	0100	1	5
5	0101	2	7
6	0110	2	9
7	0111	3	12
8	1000	1	13
9	1001	2	15
10	1010	2	17

With J is straightforward $s(n)$. We use the verb `#:`

```
#: 16

1 0 0 0 0
```

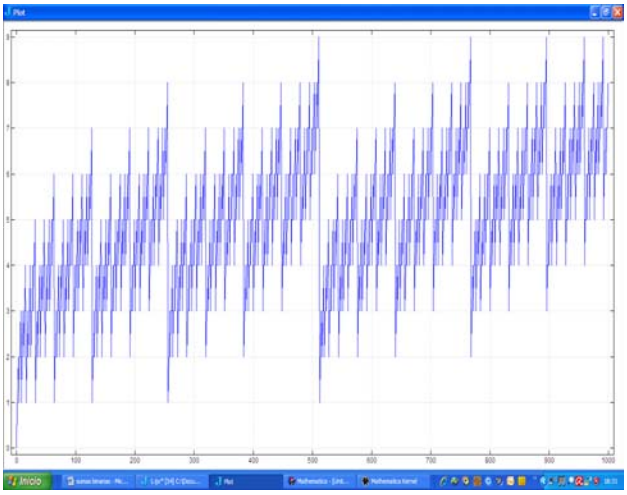
The verb +/ is useful to sum all elements of a list , which applied to the previous binary expansion gives us the desired value of $s(n)$.

```
+/#: 16

1
```

Using plot utility is easy to visualize the behavior of the function $s(n)$.

```
Load ‘plot’

Plot +/|:#: i.1000
```

Note that we have made use of the verb `|:` to sort the list resulting from applying `#:` to `i.`1000. The interesting behavior of $s(n)$ shown in the figure below .



The sequence $s(n)$ corresponds to A000120-OEIS

We are however now interested in the behavior of the sums

$$S(n) = \sum_{s=0}^{n} s(n).$$

We do:

```
+/\+/|:#:i.16

0 1 2 4 5 7 9 12 13 15 17 20 22 25 28 32
```

Consider the case where $S(N)$ is a power of 2, , $\mathrm{N} = 2^{\mathrm{k}}, \mathrm{k} = 0,1, \ldots$
You can see that:

$$S(2^k) = \frac{k\,2^k}{2},$$

Can also be written as

$$S(N) = \frac{\log N}{2 \log 2} \text{ for } N = 2^k.$$

The behavior of $S(N)$ is more complicated when N is not a power of 2. It is not strange to assume that:

$$S(N) \sim \frac{N \log N}{2\, log 2}\ if\ \ N \to \infty.$$

In fact one can show that

$S(N) = \frac{N \log N}{2 \log 2} + O(N), N \to \infty.$

Let us show you a formula for $S(N)$ valid for all N. The formula is based on a function, now called Takagi function, that is continuous but not differentiable.

we defined previously the "tent" function:

$$g(x) = \begin{cases} \frac{1}{2} x \ if \ 0 \leq x \leq \frac{1}{2} \\ \frac{1}{1}(1-x) if \ \frac{1}{2} < x < 1 \end{cases}$$

Takagi function is:

$h(x) = \sum_{r=0}^{\infty} \frac{1}{2^r} g(2^r \mathrm{x})$

H. Delange showed that:

$$S(N) = \frac{N \log N}{2 log 2} + N\mathcal{F}(\frac{log N}{\log 2})$$

and

$\mathcal{F}(l) = \frac{1}{2}(1-l) + 2^{1-l} h(2^{l-1}), 0 \leq l \leq 1.$

The function $\mathcal{F}(l)$ is the Delange. function

J-Corner

We start by defining the function tent using control structures, ie without tacit programming: Cliff Reiter

```
g1=: 3 : 'if. y<: 0.5 do. -:y else. -:-.y end.'"0
```

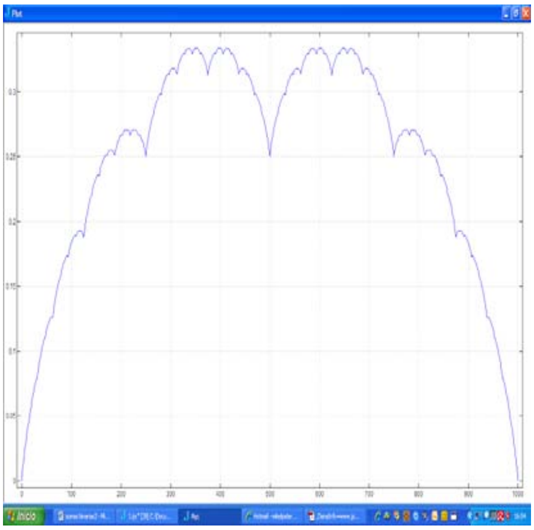
Using `@:`

```
g2=:                                     -:`(-:@-.)@.(>&0.5)"0
plot (;g2) 0.01*i.101
```

Using absolute values and arithmetic

```
g3=:0.25&*@:-.@:+:@:|@:(-&0.5)
```

```
g=:-:@:|@:(1&|&.(0.5&-))


h=:1:0("0)
r=.2^i.m
+/(g y*r)%r)

plot 100 h  0.001*i.1001
```



The code for Delange function is

```
F=: -:@-.+(%~ 100 h)@:(2 ^ -.)

plot F  0.001*i.1001
```

Ric Sherlock provides a tacit approximation to the "tent" function:

```
tent=:0:`-:`(-:@-.)`0:@.(0 0.5 1&I.)"0
```

Then we will consider exponential sums of the form

$E(N) = \sum_{n=0}^{N-1} 2^{s(n)}$,

In J can be expressed as

```
+/\(2^+/|:#:i.16)
```

1 3 5 9 11 15 19 27 29 33 37 45 49 57 65 81

This sequence is A006046-OEIS

If $2^k = N$ Then $E(2^k) = 3^k$ for $k = 0,1,\ldots$

Similar to as seen above, we can write

$E(N) = N^{\theta}$, para $N = 2^k$, donde $\theta = \frac{\log 3}{\log 2}$.

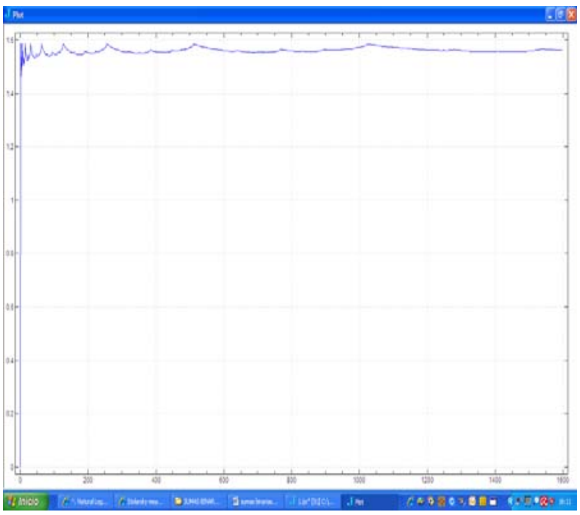
We can asume that= N^{θ} but reality is more complicated (***Stolarsky***):

$\frac{1}{3} < \frac{E(N)}{N^{\theta}} < 3.$

With J we can can calculate θ assuming that $E(N) = N^{\theta}$.

```
E=.+/\(2^+/|:#:i.1600)

N=. i.1600

Load ‘plot’

plot(^.E)%(^.N+1)
```

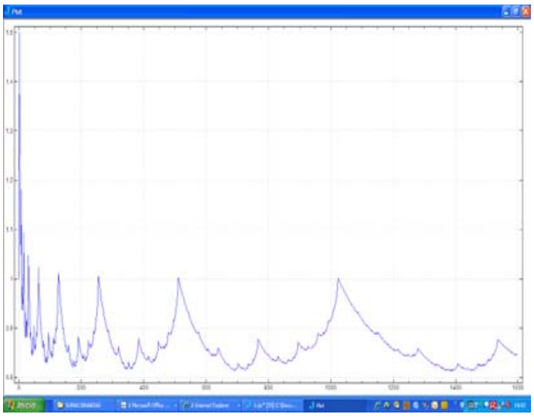
Next figure shows the relation $\frac{\ln E(N)}{\ln N}$.



consider the quotient graph

$\frac{E(N)}{N^{\theta}}$ and $\theta = \frac{\log 3}{\log 2}$

```
f=.+/\(2^+/|:#:i.1600)

n=. i.1600

fi=. (^.3)%(^.2)

nn=. n^fi

F=.f%(nn+1)

plot F
```



There is a relationship between the sums we are dealing and Pascal's triangle.

$2^{s(k)}$ is equals to the number of odd binomial coefficients of the form $\binom{k}{j}, 0 \leq j \leq k$

As a result of this, the number of odd elements of the first n rows is:

$$E(N) = \sum_{n=0}^{N-1} 2^{s(n)}$$

Acknowledgements

Thanks to Cliff Reiter, Ric Sherlock and J community for help with J-code.

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- The title page should contain (1) title of article (do not use abbreviations); (2) authors' names; (3) authors' affiliations, including adequate postal addresses;

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- The abstract should be and precise, but with enough information for the reader to understand the main points highlighted in the paper without having to read the whole article.
- List of not more than eight keywords.

The Text

- There is no artificial limit to the length of a paper, but letters to the Editor should preferably be restricted to 2 printed pages in order to speed up the refereeing process.
- The words, "**section**"/"**sections**", "**figure/figures**" and "**equation**"/"**equations**", should be abbreviated as "**Sec./Secs.**", "**Fig./Figs.**", and "**Eq./Eqs.**", respectively, whenever they occur within a sentence, e.g., Eq. (24), Eqs. (7)-(10), Fig. 11, Figs. 12-13, Sec. 5. They should, however, be spelled out in full when they occur at the beginning of a sentence.
- **Number sections, figures** and **tables** using **arabic numerals**. Subsections should be numbered using the decimal system (e.g. "3.2" means subsection 2 under section 3).
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- **Notations** must be legible, clear, compact and consistent with standard usage. All nonstandard symbols must be defined when first mentioned and at all subsequent times when confusion might arise. Distinction should be made between capital and lower cases; between the letter O and zero (0); between the letter I and the number 1 and prime; between k and kappa. A vector will be printed boldface, and to indicate this, the letter should be underscored with a single wary line.

- Cite in **numerical order** every reference, figure and table. (Order of mention in text determines the number given to each). Use Arabic numerals in superscript to cite references.

Acknowledgements

- As a footnote to the text.

References

- Place immediately after text.
- **Number the references** in order of which they are mentioned in the text.
- Abbreviate titles of journals according to the style used in the AIP Style Manual.
- Follow the correct form of citing references

Tables

- Place each table on a new page.
- Give each table a number (in arabic numerals) and a title. Number tables in order of which they are mentioned in the text.
- Provide a footnote to each table - identifying in alphabetical order all abbreviations used.

Figures

- Figures should be professionally drawn; freehand lettering is unacceptable.
- Use black ink for all charts and line drawings. Make decimals, broken lines, etc. strong enough for reproduction.
- All essential details must be legible after a reduction of at least 50%.
- Use arrows to designate special features.
- Number figures in order of which they are mentioned in the text.

Appendices

- Appendices, if any, will be placed at the end of the paper, after the list of references. Subsections within the Appendix will be numbered as A.1, A.2, etc.

Next Issue:

Focus Issue. Special number about mathematical roots of J. Authors are invited to submit papers for this special number.