2. John Napier and Logarithms

> … and ever, and ever, the logarithmic tables be they corrected, for by the error of some calculator the vessel often splits upon a rock that should have reached a friendly shore … . Henry Thoreau, *Walden*.

Introduction

The invention of logarithms is attributed to the sixteenth-century Scottish nobleman John Napier who published the first logarithm table in 1614. However, replacement of multiplication by addition and division by subtraction, the basic idea in the use of logarithms, may be found in the writings of Archimedes in the third century B.C. In the *Sand Reckoner* he found an upper bound on the number of grains of sand required to fill a sphere large enough to contain the universe as it was known to the Greeks. He gave this estimate, which we would write as 10^{63}, as ten million units of the eighth order of numbers, and remarked when defining the various orders of numbers that the addition of the orders of numbers corresponded to their multiplication. In the sixteenth century the French mathematician Nicolas Chuquet in his *Triparty en la sciences des nombres* gave a table of the powers of 2 from 0 to 20 and remarked that sums of the indices corresponded to the products of the corresponding powers. Michael Stifel in *Arithmetica integra* in 1544 extended this idea to negative and fractional exponents.

In this chapter we will consider the life and work of John Napier including both the development of his logarithm tables and his mechanical calculating device known as Napier's rods, the modification and extension of logarithm tables by Napier's contemporaries, and the development of the slide rule which may be considered as a mechanization of logarithms.

John Napier of Murchison

John Napier, the eighth Baron of Merchiston, was born at Merchiston near Edinburgh in 1550. At the early age of 13 he matriculated at the University of St. Andrews. There is no record that he ever graduated, and he is believed to have left St. Andrews to complete his education on the Continent. He returned to Scotland by 1571, and the next year he married for the first time. His first wife died in 1579, having borne him a son and a daughter. A few years later he remarried, and by this marriage had five sons and five daughters. His second son, Robert, became his literary executor.

Napier, an ardent Protestant, was much involved in the affairs of the Church of Scotland. In 1588 he was chosen by the presbytery of Edinburgh to be one of the Commissionaers of the General Assembly. In 1593 he was appointed to a committee to discuss with the King the punishment of rebellious Popish earls and the safety of the Church. At about this time Napier published what he always considered to be his most important work, *A Plaine Discovery of the whole Revelation of Saint John*, which he wrote in English instead of Latin in order that "hereby the simple of this Lland may be instructed". It went through five editions in English, as well as nine in French, four in German and three in Dutch. This work is still considered of importance in the history of Scottish theological literature.

In addition to his theological and mathematical work Napier was also interested in devising instruments of war. His inventions included a mirror for burning ships at a distance, an artillery piece which destroyed everything in the arc of a circle, and an armed vehicle which may have been the predecessor of the tank. This last device was successfully demonstrated with some Scottish sheep and cattle becoming what may have been the first casualties of tank warfare.

Napier died in 1617 and was buried beside St. Cuthbert's Church, Edinburgh where he is said to have worshipped "Sunday after Sunday". An inscription on the church tower reads "Near this spot was laid to rest John Napier of Merchiston, who gained for himself the imperishable memory of future ages by his wonderful discovery of logarithms."

The lecture given in 1914 by E. W. Hobson, Sadlerian Professor of Pure Mathematics at Cambridge University, on the 300th anniversary of the publication of John Napier's historic table of logarithms begins as follows:

> In the present year there will be held a celebration, under the auspices of the Royal Society of Edinburgh, of the tercentenary of one of the great events in the history of Science, the publication of John Napier's "Mirifici Logarithmorum Canonis Descriptio," a work which embodies one of the greatest discoveries that the world has seen. The invention of Logarithms not only marks an advance of the first importance in Mathematical Science, but as providing a great labour-saving instrument for the use of all those who have occasion to carry out extensive numerical calculations it can be compared in importance only with the the great Indian invention of our system of numeration.

Napier's logarithms

In about 1594 Napier became interested in methods of lessening the burden of the calculations in spherical trigonometry performed by astronomers, a matter of concern to him which may be indicated by the following comment:

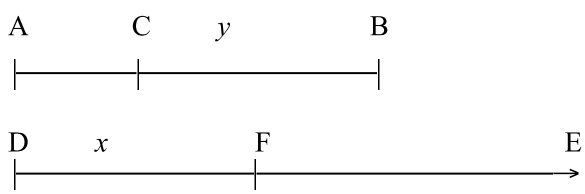
> There is nothing more troublesome in mathematics than the multiplications, divisions, square and cubic root extractions of great numbers which involve a tedious expediture of time, as well as being subject to "slippery errors".

The idea to replace multiplication and division by addition and subtraction may have come in part from the correspondence discussed above between geometric and arithmetic progessions. He was undoubtedly aware also of the so-called prosthaphaeretic rules such as

$$\sin(x)\sin(y) = \tfrac{1}{2}\,[\cos(x - y) - \cos(x + y)]$$

where products of two sines or cosines may be expressed as the sums or differences of these functions. After many years of labour Napier published in 1614 his *Mirifici logarithmorum canonis descriptio*, a small work containing 57 pages of explanatory matter and 90 pages of tables. It has been regarded in the history of British science as second only to Newton's *Principia*. It contains the first announcement of logarithms, the first table of logarithms, and the first use of the word "logarithm" meaning "ratio number", a word coined by Napier. The *Descriptio* contained no account of how his logarithms were computed. Napier did write such an account, *Mirifici logarithmorum canonis constructio*, which was published posthumously by his son Robert in 1619. An English translation of the *Constructio* was published in Edinburgh in 1889.

Whatever were the origins of Napier's ideas about logarithms, his final concept was based on a dynamical geometric argument. Consider a line segment AB of finite length and another line segment DE of semi-infinite length, and consider points C and F starting at the same time from A and D, respectively, and moving along their respective line segments:



The velocity of C is always equal to the distance CB, while that of F remains equal to its initial velocity which is the same as the initial velocity of C. Then Napier defined DF to be the logarithm of CB, i.e.,

$$y = \text{Nap log } x \ .$$

Napier took the length AB to be 10^7 since the best sine tables then available were to seven decimal places. Unlike present definitions of logarithms, Napier's gave a value of 10^7 for the logarithm of 1, and furthermore Napier's logarithms increased in value as the argument decreased. The tables in the *Descriptio* gave the values of the logarithms of the sines of angles from 0 to 90 degrees at intervals of one minute to seven or eight decimal places. It has been often said that Napier's logarithms were natural or "Naperian" logarithms to the base $e = 2.71828\ldots$, but this is incorrect although it may be shown that

$$\text{Nap log } x = 10^7 \log_{1/e} (x/10^7) \ .$$

The publication of Napier's *Descriptio* in 1614 immediately attracted the attention of Henry Briggs, then professor of geometry at Gresham College, London and later the first Savilian professor of geometry at Oxford. Briggs wrote to his friend James Ussher about Napier's work as follows:

> Napper, lord of Markinston, hath set my head and hands a work with his new and admirable logarithms. I hope to see him this summer, if it please God, for I never saw a book which pleased me better or made me more wonder.

Briggs began work on his own table of logarithms using what we would now term a base of 10 and with the logarithm of 1 being 0. Briggs made the four-day journey from London to Edinburgh in the summer of 1615. It was recorded that when the two men first met they gazed at each other in admiration for a quarter of an hour without speaking. Briggs and Napier agreed both on the use of a base of 10, the value of 0 for the logarithm of 1, and also on the value of 1 for the logarithm of 10. Briggs visited Napier a second time during the summer of 1616 taking with him his partially completed new table of logarithms, and would have made a third journey the following summer if Napier had not died that spring.

Further development of logarithm tables

In the year of Napier's death Briggs published his *Logarithmorum chilias prima* giving the logarithms to base 10 of the first 1000 positive integers to 14 decimal places. In 1624 he published *Arithmetica logarithmica* with the logarithms of the integers from 1 to 20000 and from 90000 to 100000 and an 88-page introduction describing the method of calculation and the applications of logarithms. The gap between 20000 and 90000 was filled in 1628 when Adrian Vlacq of Gouda in the Netherlands published a table of logarithms of the integers between 1 and 100000 to 10 decimal places. Although Vlacq had computed 70000 of these values copying 30000 from Briggs, he called his table a second edition of the *Arithmetica logarithmica*. Vlacq's tables formed the basis of all of the many tables of loagrithms published up until the early years of the twentieth century.

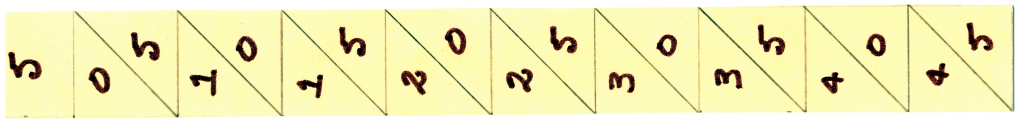
The only person who may be considered to have a rival claim for the invention of logarithms is Jobst Bürgi, a Swiss watchmaker, who may have had the idea as early as 1588, a few years before Napier began his work. In 1620 he published in Prague his *Arithmetische und geometrische Progress-Tabulen*, a work differing from Napier's mostly in terminology and in numerical values used rather than in fundamental principles.

Napier's rods

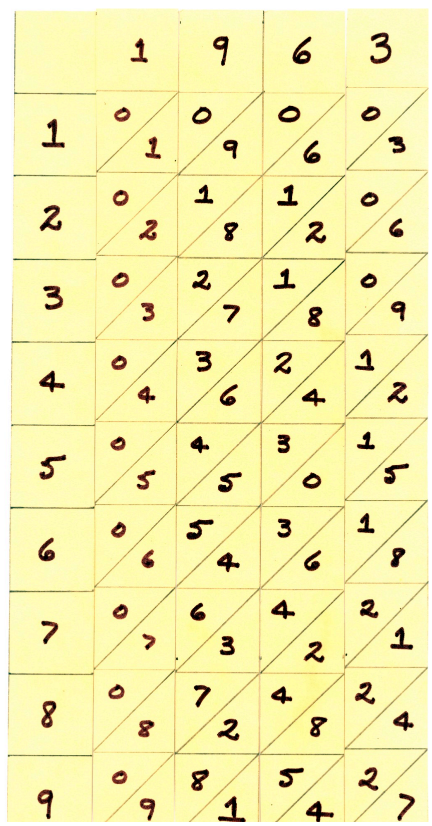
In 1617, the year of his death, Napier published his *Rabdologia* which described a mechanization of the gelosia method of multiplication described in the last chapter. A translation in 1667 of this work from

Latin into English incorrectly translated "rabdologia" as a "collection of bones" rather than as a "collection of rods" and his device became to be known as either "Napier's rods" or "Napier's bones".

Napier's rods consisted of 10 rectangular rods giving the multiples from 1 to 9 of the digit shown at the top of the rod. The following is a rod showing multiples of 5:



As an example of how the rods are used consider the example in the last chapter of finding the product of 472 and 1963. Rods with the digits 1, 9, 6 and 3 at the top are laid side-by-side with a rod bearing only the digits 1, 2, ..., 9 on the left:



Then from the row labelled "4" find the diagonal sums obtaining 7852 giving the product of 4 times 1963. Similarly from rows labelled "7" and "2" obtain the sums 13741 and 3926, respectively. Finally align these sums as

    3 9 2 6
  1 3 7 4 1
  7 8 5 2

and find their sum, 926536, which is the desired product of 472 and 1963.

Napier's rods were made of strips of wood or were of square cross section with a different multiples on the four sides. Some sets were mounted in boxes so that they could be turned by handles. Napier's rods were used extensively in Europe and were introduced into China and Japan.

Slide rules

Soon after the first publication of logarithmic tables it was realized that their use could be mechanized so that calculations could be done without their direct use. In 1620 Edmund Gunter made scales with one having the digits from 1 to 10 in positions marked according to their logarithms and another with positions corresponding to the logarithms of the trigonmetric functions. Lengths on the scales were added or subtracted, i.e., the corresponding numbers multiplied or divided, by means of a pair of dividers. In 1628 Edmund Wingate published in London a booklet describing a double scale with numbers marked on one side and the corresponding logarithms on the other. Two years later he published *Natural and Artificial Arithmetic* giving a description of a sliding scale consisting of two scales with one sliding along the other and held together by hand.

The idea of a sliding scale appears to have been developed independently by William Oughtred, an English clergyman who devoted much of his time to the study and teaching of mathematics, who also developed a circular rule consisting of concentric logarithmic scales to which were attached two radial pointers. Oughtred's notes, written in Latin, were translated and published in 1632 by a former pupil William Foster, a teacher of mathematics in London. In his dedication to Oughtred's work Foster attributes to Oughtred the following reason for not publicizing his work:

> … the true way of Art is not by Instruments, but by demonstration: and that it is a preposterous course of vulgar teachers, to begin with Instruments and not with the Sciences, and so instead of Artists to make their Schollers only doers of tricks, and as it were Jugglers … the use of Instruments is excellent, if a man be an Artist …

Oughtred's claim to the invention of the circular slide rule was disputed by one of his pupils, Richard Delamain, who allegedly copied his work and claimed it as his own. Delamain said, incidentally, that his rule was "fit to use … as well on Horse back as on foot".

There was a continuous development of slide rules from the time of Gunter and Oughtred until the twentieth century. The first slide rule in which the slide worked between parts of a fixed stock appeared in 1654. Isaac Newton suggested the use of a cursor to assist in reading the numbers on the scales as early as 1657, but one was not developed for another one hundred years. Many slide rules were developed for special calculations such as those arising in navigation and in volume calculations. The slide rule in the form it was used until it was replaced by electronic calculators in the 1970s was developed by the French army officer Amédée Mannheim in the middle of the nineteenth century.