

3. Charles Babbage and his Engines

“I wish to God these calculations had been executed by steam.” Charles Babbage. “It is quite possible.” John Herschel.

Charles Babbage

Charles Babbage was born on Boxing Day of 1791 in London, although several early accounts of his life and work state that he was born the previous year in Totnes, Devonshire. His father was a wealthy banker, and Charles received his elementary education at a boarding school. His father then decided that his son should go to Cambridge and asked a Cambridge tutor to give his son advice on what he might expect. In later years Babbage recalled that the advice could be summed up quite simply by saying that he was told “not to purchase his wine in Cambridge”.

In 1810 he entered Trinity College, Cambridge, but before graduation he transferred to Peterhouse as he did not wish to graduate below his friends at Trinity. Included amongst his friends were John Herschel, the son of the discoverer of the planet Uranus who became a distinguished astronomer himself, and George Peacock who became Dean of Ely. These three, while undergraduates, resolved to “do their best to leave the world wiser than they found it”, and founded the Analytical Society to further their purposes. One of their more notable activities was to promote the notation of Leibniz for the differential calculus in preference to that of Newton, or in Babbage’s words “the principles of D-ism as opposed to the Dot-age of the University”. They also produced a translation of *Traité du calcul différentiel et du calcul intégral* by the French mathematician S. F. Lacroix and included explanatory notes to accompany some sections. This book became the standard English-language calculus text for the first part of the nineteenth century.

After graduating from Cambridge Babbage returned to London where he spent the rest of his life engaged in study and research having inherited about 100,000 pounds from his father. He also led an active social life, both attending evening parties hosted by London society and giving parties, invitations for which were eagerly sought after. He became a Fellow of the Royal Society in 1816, and in 1828 was appointed Lucasian Professor of Mathematics at Cambridge, a position once held by Isaac Newton. He held this appointment for eleven years but rarely visited Cambridge and never gave a lecture. Babbage received many European honours and was a member of at least 15 European scientific societies. In addition to his lifelong work with his computing engines, he busied himself with other problems and was a prolific writer. He was one of the earliest proponents of what is now known as operational research, and because of his analysis of the British postal system the British Post Office was re-organized under Sir

Rowland Hill. He also invented the “cow catcher” attached to the front of locomotives, and was influential in the decision to adopt the broad rather than the narrow gauge railroad track.

Babbage died in 1871 at the age 79 and was mourned by only a few personal friends who survived him.

The method of differences

The method of differences, although no longer used, was employed in the construction of astronomical and navigation tables as a means of replacing the difficult and error-prone operations of multiplication and division by the much simpler operation of addition. To illustrate the method consider the following table the first two rows of which give the first few values of the second-degree polynomial

$$T(x) = x^2 - x + 41$$

for positive values of x :

x	1	2	3	4	5	6	7	...
$T(x)$	41	43	47	53	61	71	83	...
D_1		2	4	6	8	10	12	...
D_2			2	2	2	2	2	...

The third row gives the first differences of the function $T(x)$ which are defined as the differences of successive values of the function $T(x)$. The fourth row gives the second differences of $T(x)$ which are the differences of successive values of the first differences. Since the second differences have the constant value 2, successive functional values may be calculated by first extending the row of first differences by the addition of the constant 2 and then adding these first differences in turn to the functional values in the second row. Thus the next first difference in the table is $12 + 2 = 14$ and the next functional value is $83 + 14 = 97$ which is the value of $T(x)$ evaluated for $x = 8$ or $8^2 - 8 + 41$.

The second differences in the above example were constant since $T(x)$ is a polynomial of degree 2, i.e., the highest order term is x^2 . In general, a polynomial of degree n in which the highest order term would be x^n would have constant n th-order differences. Thus the table of a polynomial of any degree could be calculated by the method of differencing thus avoiding any multiplication. The method was of interest in Babbage’s time since it could be used in the calculation of logarithmic and trigonometric tables by approximating the functions by appropriate polynomials.

The function $T(x)$ is due to the Swiss mathematician Leonhard Euler (1707 – 1783) and its values for positive integral values of x less than 41 are prime. These numbers are known as Euler numbers. We shall mention this function again in the last section in a brief discussion of the use of **J** in working with prime numbers.

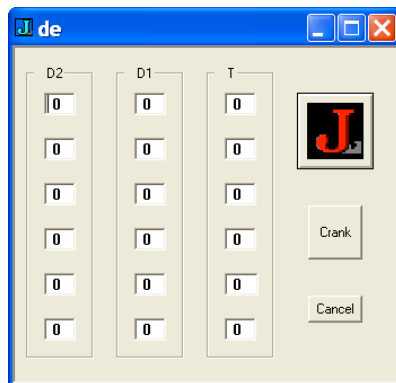
The Difference Engine

There are several accounts of how Babbage originally conceived the idea of constructing a machine that would perform calculations. Possibly the most reliable is that he and Herschel were proofreading some astronomical tables which contained many errors, and their exasperation led to an exchange similar to that quoted at the head of this chapter. In any event Babbage became preoccupied with the problem and was determined to build a machine that could calculate a sequence of values of a polynomial by the method of differences.

Babbage first built a small Difference Engine that could accommodate first- and second-order differences to six decimal places. The functional values and the differences were each represented by a vertical column of toothed wheels with each wheel representing a digit. (Negative numbers were represented by complements.) The machine was operated by an horizontal crank at the top, and with each turn of the crank the constant second-order difference was added to the first-order difference giving the next first-order difference which was then added to the functional value giving the next functional value. He presented a paper on this machine to the Astronomical Society in 1822 and the following year received the Society's Gold Medal, the first to be presented by the Society. Up to that time Babbage had financed his work entirely himself. However with the support of the Society he then applied to the Government for funds to develop a larger machine, referred to now as Difference Engine No. 1, that could accommodate sixth-order differences to twenty figures. The following year the Chancellor of the Exchequer agreed to advance 1500 pounds and Babbage agreed to put up between 3000 and 5000 pounds of his own money. As Babbage worked on his machine further funds were forthcoming both from the Government and from Babbage himself. However the machine was never completed and when work finally terminated in 1842 the Government had contributed a total of 17,000 pounds and Babbage a further 20,000 pounds. Parts of the Difference Engine may be seen today in the London Science Museum.

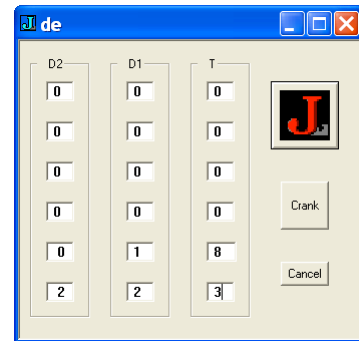
A simulator for the Difference Engine

A simulator has been written in the **J** language for the first Difference Engine that Babbage



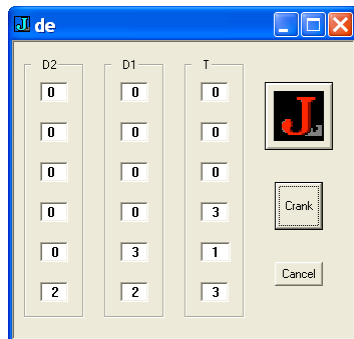
constructed that would accommodate second-order differences. The first figure shows the Windows form for the simulator in its initial configuration. Initial values of the first- and second-order differences are entered manually into the Windows form. Each click of the button marked "Crank" corresponds to one turn of the crank of the Difference Engine and results in the next value for each of the first-order difference and functional value to be calculated and displayed. The second figure shows the simulator set up for the calculation of

successive values of Euler's quadratic discussed in an earlier section. The final figure shows the simulator after 10 turns of the "crank" resulting in a functional value of 313 which corresponds to the value of $T(x) = x^2 - x + 41$ for $x = 17$.



The Analytical Engine

Although work continued on the Difference Engine until 1842,



Babbage lost interest in it ten years

earlier. At that time he conceived a plan for a much grander machine, which he called an Analytical Engine, and which would be a truly automatic computer. The Analytical Engine was to have the following five components: a store for holding numbers to be operated on; an arithmetical unit, which Babbage called a "mill", for performing the arithmetic operations; a control unit for ensuring that the operations were

performed in the correct sequence; an input device for providing the machine with data; and an output device for displaying the results of the calculation. Numbers were to be represented as they were represented in the Difference Engine by columns of wheels with one column for each number and one wheel for each digit. The store was to accommodate 1000 fifty-digit numbers. Transfer of data between store and the mill was to be by an elaborate system of gears and rods. Control of a calculation was to be accomplished by means of punched cards similar to those used to control the operation of the Jacquard loom. There were to be two types of cards to specify the type of operation to be performed in the mill and variable cards to control the transfer of numbers between the store and the mill. Input was to be accomplished by manually setting the wheels of the store although tables of mathematical functions were to be supplied on punched cards. Output could be printed on paper, punched on cards, or cast in molds from which printers' blocks could be made. On aspect of the operation of the Analytical Engine to which Babbage devoted a considerable attention was the handling of carry digits which could arise in addition. He eventually developed an anticipatory carry in which all of the carry digits could be handled simultaneously.

Unfortunately Babbage wrote no complete account of the Analytical Engine. However in 1840 he gave a series of public lectures in Turin which were written up by a young engineer L. F. Menabrea, who became Prime Minister of Italy in 1867, and published in a Swiss Journal in 1842. A year later they were translated into English and extensively annotated by Lady Lovelace, daughter of the poet Lord Byron, and published in *Taylor's Scientific Memoirs*. The paper is mainly concerned with the

programming of the Analytical Engine, and several examples of programs are discussed in considerable detail.

Lady Lovelace, born Ada Augusta Byron, had considerable mathematical talent, a characteristic she shared with her mother. Her mathematical interests were encouraged by her mother and she studied for some years with the noted mathematician Augustus De Morgan and his wife. She became interested in Babbage's work in 1834 at the age of nineteen when she attended a lecture by Dionysius Lardner on the Difference Engine. This paper which was published in the *Edinburgh Review* gave a detailed description of the Difference Engine and its possible applications. In 1835 Ada Byron married William, Baron King who was created the first Earl of Lovelace in 1838. Lord and Lady Lovelace became friends of Babbage, and she was able to follow his work closely. Lady Lovelace died of cancer in 1857 at the young age of 37.

Other Difference and Analytical Engines

Dionysius Lardner's paper on the Difference Engine stimulated a Swedish engineer, George Scheutz, to build a similar machine that could accommodate fourth-order differences to fourteen figures. A copy of this machine was made for the British Government and was used in the Office of the Registrar General for the preparation of life tables. Several types of Difference Engines were built in the early 1900s, some of them being adaptations of existing calculating devices.

The design of difference and analytical engines was considered in great detail by a little-known Irishman, Percy E. Ludgate, who died in 1922 at the age of 39. Ludgate was employed as an auditor in Dublin, and worked, apparently alone, on his calculating machines as a hobby. In his one major paper published in 1909 in the *Scientific Proceedings, Royal Dublin Society* he pays tribute to Babbage, but said that he was unaware of his work until he had completed the first design of his machine. Ludgate proposed a store of 192 twenty-five-digit numbers each of which was to be stored in a "shuttle" where the digits and sign were to be represented by protruding rods. Numbers were to be brought into the arithmetic unit by appropriate rotation of the shuttle. Multiplication was to be done by first finding products of pairs of digits logarithmically and then adding the partial products. The sequence of calculations was to be controlled by a "formula paper" in which each row of perforations represented the operation to be performed and also the location in the store of the two operands and the result. Conditional branching was to be accommodated by shifting backwards or forwards on the tape. Operations such as division and the calculation of logarithms were to be performed by sequences of instructions on permanent cylinders.

There is no indication that Ludgate ever constructed either a difference or an analytical engine. None of Ludgate's work appears to have survived except for two published papers, the one already referred to and a paper in the *Handbook* of the Napier Tercentenary Celebration which gives an account of Babbage's Analytical Engine and a brief mention of his own work.

Possibly the most interesting continuation of Babbage's work is the construction by the London Science Museum of Difference Engine No. 2 from Babbage's plans for his uncompleted Difference Engine No. 1. This work which was begun in the second half of the 1980s was undertaken in part to see if engineering practice in Babbage's time would have been sufficiently precise for the construction of a working machine and partly as a most fitting tribute to Babbage on the 200th anniversary of his birth on December 26, 1991. A fascinating account of this work which involved "bankruptcy, funding crises, mishaps and the politics of any major engineering project" has been told by Doron Swade, an Assistant Director and Head of Collections the Science Museum, who was in charge of the project. (Swade's book was published in Great Britain with the imaginative main title of *The Cogwheel Brain* and a couple of years later in America more prosaically as *The Difference Engine*.) A better introduction to the life and times and work of Charles Babbage would be difficult to find.

Prime numbers and coffee tables

The example used to illustrate the method of differences was the polynomial

$$T(x) = x^2 - x + 41$$

which as was mentioned gives prime numbers for positive integral values of x less than 41. In this section we shall use the introduction of this polynomial to make a few remarks about prime numbers and conclude with what might be called a decorative application of primes. However first we shall introduce two **J** verbs which are useful in dealing with prime numbers.

The expression $p: i$, where $p:$ is the monadic verb *primes*, gives the i th prime (in zero-origin indexing). For example, $p: 5$ is 13, and $p: i. 10$ is the list 2 3 5 7 11 13 17 19 23 29 of the first 10 primes.. The monadic verb *prime factors* $q:$ gives the list of prime factors of its positive integer argument. For example, $q: 126$ is 2 3 3 7, and $q: 97$ is 97 since 97 is prime. (There is a dyadic form of $q:$, *prime exponents*, which need not concern us here.) The following dialogue gives some examples of the use of these verbs:

```

prime=: 1: = [: # q:      NB. Prime test
prime 126
0
prime 97
1
prime"0 (126 97)
0 1
Q1=: 3 : '(y. * y.) +41 - y.'  NB. Euler's quadratic
x=. >:i. 7

```

```

|: x,. Q1 x          NB. |: is monadic Transpose, i.e.,
1 2 3 4 5 6 7        NB. rows and columns are interchanged
41 43 47 53 61 71 83
prime"0 Q1 x
1 1 1 1 1 1 1
+/prime"0 Q1 x        NB. Number of primes in the list Q1 x
7
+/prime"0 Q1 >: i. 40  NB. All values of Q1 x for x <= 40
40                    NB. are prime
Q1 41
1681
prime Q1 41
0                    NB. Q1 41 is not prime
([ , prime] Q1 41
1681 0
+/prime"0 Q1 >: i. 1000  NB. Number of prime values of Q1 x
581                    NB. for x <= 1000

```

A method of generating prime numbers without using the verb `p:` is given by the statement

```
Primes=: (-. x e. ,x */x) # x=. }.>:i.N
```

which gives a list of primes less than or equal to N which may be read simply as “The list of primes less than or equal to any non-negative integer N may be found by selecting those integers in the list $x=. 2, 3, \dots, N$ which do not appear in the multiplication table for x .” We may note that for N equal to 0 or 1 the list `Primes` is empty.

An interesting “recreational” example of prime numbers was provided some years ago by Stanislaw Ulam, a mathematician at the Los Alamos Scientific Laboratory. At a conference he was idly doodling during what he termed a “long and very boring paper” and observed that if the positive integers were first arranged in a rectangular spiral then the prime numbers tended to lie along diagonal straight lines. Even the following small example of a rectangular spiral array gives a little indication of the grouping of the primes along diagonal lines:

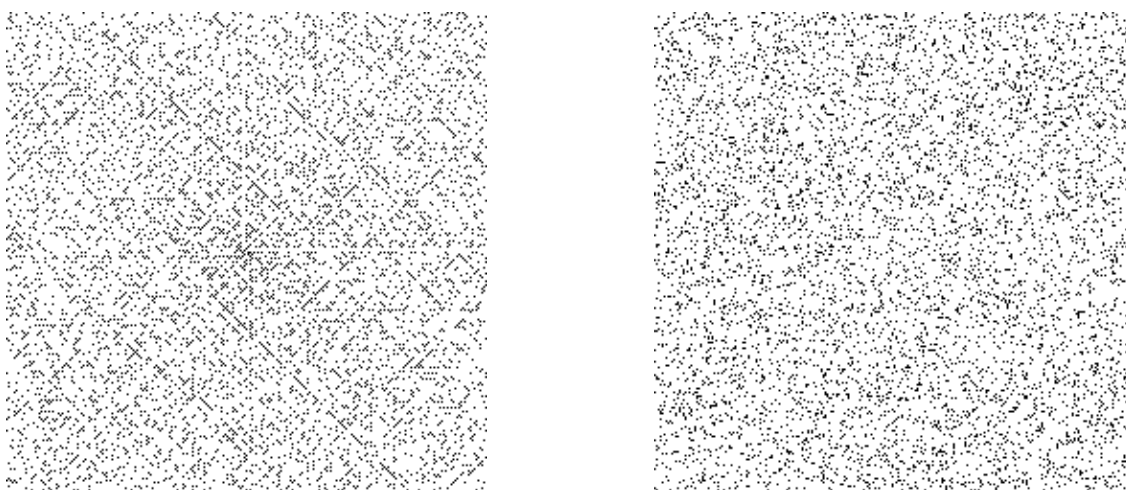
```

100 99 98 97 96 95 94 93 92 91 90 89
101 64 63 62 61 60 59 58 57 56 55 88
102 65 36 35 34 33 32 31 30 29 54 87
103 66 37 16 15 14 13 12 11 28 53 86
104 67 38 17 4 3 2 1 10 27 52 85
105 68 39 18 5 6 7 8 9 26 51 84
106 69 40 19 20 21 22 23 24 25 50 83
107 70 41 42 43 44 45 46 47 48 49 82
108 71 72 73 74 75 76 77 78 79 80 81

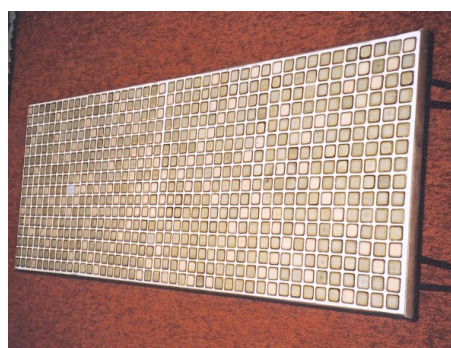
```

This property was investigated further using the computer at Los Alamos and a table of the first 90 million prime numbers stored on magnetic tape. Images were produced for various spiral arrangements of consecutive integers including the integers from 1 to about 10 000 and from 1 to about 65 000. This experimental work provided some rather striking further evidence of Ulam's conjecture. (There is a large painting illustrating this spiral arrangement of primes in a lecture room in the School of Mathematics at the University of Newcastle-upon-Tyne which was a gift of a visitor as an expression of appreciation for his visit.)

The figure on the left below, which was constructed very simply in **J**, gives the image produced by a spiral arrangement with 251 rows and 251 columns of the integers from 1 to 63001. There are 6320 primes represented in the above array. The second figure gives the image produced for a random rather than a spiral arrangement of integers with the primes marked as in the previous figure.



The spiral arrangement of the integers with the primes marked can make an attractive tiling pattern as shown by the accompanying picture. The coffee table has 16 rows and 43 columns of one-inch tiles



considered to be a spiral arrangement of the first 16×43 or 688 positive integers. One colour of tile was used for tiles corresponding to primes, another colour for those corresponding to non-primes, and a third colour for the tile corresponding to the integer 1. The number of prime tiles required is

```
+ /prime"0 (>:i. 688)
```

which is equal to 124 so that the number of non-prime tiles greater than 1 is 563.

A spiral arrangement of integers may be found by the defined verb *Spiral* defined by the following sequence of definitions:


```

pn=: >:@i.
OneTurn=: [ , "2 |.@|:@]
rows=: 1: >. {.@($@])
Wind=: [ `((rows@[{.]) OneTurn [) Wind rows@[{.])@.(0:<#@])
Spiral=: (i. @ ((-/ @ ]) , 0:)) Wind pn@(*/)@]

```

The details need not concern us except that we may note that the 9-by-12 spiral array of the integers given earlier in this section may be calculated by the expression `|:Spiral 9 12` where `|:` is the primitive monadic verb *transpose*.